Reg. No. \_\_\_\_\_\_\_\_\_\_\_\_\_



**End Semester Examination – Nov / Dec – 2019**

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| **Code :** | **19MA3004** | **Duration :** | **3hrs** |
| **Sub. Name :** | **REAL ANALYSIS** | **Max. Marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | | **Course**  **Outcome** | | **Marks** |
| 1. | a. | Prove that every pair of integers *a* and *b* has a common divisor *d* of the form *d = ax +by,* where *x* and *y* are integers. Moreover, show that every common divisor of *a* and *b* divides this *d.* | | CO1 | | 10 |
| b. | Let *A* and *B* be subsets of *R,* let *C* denote the set *C = {x+y : x ε A, yεB}.* If each of *A* and *B* has a supremum, then prove that *C* has a supremum and *Sup C = Sup A + Sup B.* | | CO1 | | 10 |
| **(OR)** | | | | | | |
| 2. | a. | State and prove the *approximation property of supremum.* | | CO1 | 6 | |
| b. | State and prove the *triangle inequality* of real numbers and hence prove *|a + b| ≥ ||a| - |b||* for any real numbers *a* and *b.* | | CO1 | 14 | |
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| 3. | a. | Prove that the set of all real numbers is uncountable. | | CO2 | 10 | |
| b. | Let F be a collection of sets. Then prove that for any set B, we have   1. . 2. . | | CO2 | 10 | |
| **(OR)** | | | | | | |
| 4. | a. | Prove that every point of a non-empty open set *S* belongs to one and only one component interval of *S.* | | CO2 | 10 | |
| b. | State and prove the representation theorem for open sets on the real lines. | | CO2 | 10 | |
|  |  |  | |  |  | |
| 5. |  | Prove that if a bounded set *S* in *Rn* contains infinitely many points, then there is at least one point in *Rn*which is an accumulation point of *S.* | | CO3 | 20 | |
| **(OR)** | | | | | | |
| 6. | a. | State and prove *the Cantor intersection Theorem.* | | CO3 | 10 | |
| b. | State and prove *the Fixed-point theorem for contractions.* | | CO3 | 10 | |
|  |  |  | |  |  | |
| 7. | a. | Let *f* and *g* be functions defined on *(a, b)* and differentiable at *c.* Then prove that *f +g, f – g,* and *f.g* are also differentiable at *c*. Also *f / g* is differentiable if *g(c) ≠0*. | | CO4 | 10 | |
| b. | State and prove *the Rolle’s theorem.* | | CO4 | 10 | |
| **(OR)** | | | | | | |
| 8. | a. | State and prove *the Cauchy condition for uniform convergence of sequence.* | CO5 | | 10 | |
| b. | Assume that *fn→f is* uniformly continuous on *S*. If each *fn*is continuous at a point *c* of *S*, then prove that the limit function *f* is also continuous at *c*. | CO5 | | 10 | |
|  | | **Compulsory**: |  | |  | |
| 9. | a. | Let *{Mn}* be a sequence of nonnegative numbers such that *0 ≤ |fn(x)|≤ Mn*for *n = 1,2,3,..,* and for every *x* in *S.* Then prove that  converges uniformly on *S* if converges. | CO6 | | 10 | |
| b. | State and prove *the Dirichlet’s test for uniform convergence.* | CO6 | | 10 | |