Reg. No. \_\_\_\_\_\_\_\_\_\_\_\_\_



**End Semester Examination – Nov / Dec – 2019**

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| **Code :** | **19MA3001** | **Duration :** | **3hrs** |
| **Sub. Name :** | **MODERN ALGEBRA** | **Max. Marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | State and prove fundamental theorem of arithmetic. | CO1 | 12 |
| b. | If  and  with then prove that | CO1 | 8 |
| **(OR)** | | | | |
| 2. | a. | State and prove division algorithm. | CO1 | 8 |
| b. | Find the gcd(12378 , 3054) using Euclidean algorithm and then write gcd as a linear combination of 12378 and 3054. | CO1 | 5 |
| c. | If is a prime and  then prove that for some  where | CO1 | 7 |
|  |  |  |  |  |
| 3. | a. | If is a prime number and  then the congruence  has exactly  solutions. | CO2 | 10 |
|  | b. | If the integer  has order modulo then  if and only if | CO2 | 10 |
| **(OR)** | | | | |
| 4. | a. | Solve the system of linear congruences and | CO2 | 10 |
| b. | State and prove Fermat’s theorem. | CO2 | 10 |
|  |  |  |  |  |
| 5. | a. | Suppose that  is a finite abelian group and  whereis a prime number. Prove that there is an element  such that | CO3 | 10 |
| b. | Let  be a group. If  where is a prime number, then | CO3 | 10 |
| **(OR)** | | | | |
| 6. | a. | Ifis a prime number and  then has a subgroup of order . | CO3 | 10 |
| b. | Prove that a group of order 72 has a non-trivial normal subgroup. | CO3 | 10 |
|  |  |  |  |  |
| 7. | a. | Let G be a group and suppose that G is the internal direct product of  Let  Prove that G and T are isomorphic. | CO4 | 8 |
| b. | Prove that every finite abelian group is the direct product of cyclic groups. | CO4 | 12 |
| **(OR)** | | | | |
| 8. | a. | Prove that a finite integral domain is a field. | CO5 | 10 |
| b. | If is a commutative ring with unit element and is an ideal of , then prove thatis a maximal ideal of if and only if  is a field. | CO5 | 10 |
|  | | **Compulsory**: |  |  |
| 9. | a. | State and prove Unique Factorization Theorem. | CO6 | 10 |
| b. | Let  be a polynomial with integer coefficients. Suppose that for some prime number ∤∤ Then prove that is irreducible over the rationals. | CO6 | 10 |