Reg. No. \_\_\_\_\_\_\_\_\_\_\_\_\_



**End Semester Examination – Nov / Dec – 2019**

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| **Code :** | **10MA302** | **Duration :** | **3hrs** |
| **Sub. Name :** | **COMMUTATIVE ALGEBRA** | **Max. Marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | Let  be a ring homomorphism such that  is a unit in for all  Then prove that there exists a unique ring homomorphism  such that | CO1 | 10 |
| b. | Let  be an - module. Show that the following are equivalent:  (i)  (ii)  for all prime ideals of ;  (iii)  for all maximal ideals of . | CO1 | 10 |
| **(OR)** | | | | |
| 2. | a. | State and Prove First Uniqueness Theorem. | CO1 | 10 |
| b. | Let  be a primary ideal in a ring  Then prove that  is the smallest prime ideal containing | CO1 | 10 |
|  |  |  |  |  |
| 3. | a. | Let  be an integral domain. Then prove that the following are equivalent:   1. is integrally closed; 2. is integrally closed, for each prime ideal 3. is integrally closed, for each maximal ideal | CO1 | 10 |
| b. | State and Prove Going-up Theorem. | CO1 | 10 |
| **(OR)** | | | | |
| 4. | a. | A module  has a composition series   satisfies both chain conditions. | CO1 | 10 |
| b. | Show that is a Noetherian -module every submodule of  is finitely generated. | CO1 | 10 |
|  |  |  |  |  |
| 5. | a. | Show that in a Noetherian ring every irreducible ideal is primary. | CO1 | 10 |
| b. | If  is Noetherian, then the polynomial ring  is Noetherian. | CO1 | 10 |
| **(OR)** | | | | |
| 6. | a. | State and Prove structure theorem for Artin rings. | CO1 | 10 |
| b. | In an Artin ring  prove that every prime ideal is maximal. | CO1 | 10 |
|  |  |  |  |  |
| 7. | a. | Let  be an integral domain. Prove that  is a Dedekind domain every non-zero fractional ideal of  is invertible. | CO1 | 10 |
| b. | Let  be a local domain. Show that  is a discrete valuation ring every non-zero fractional ideal of  is invertible. | CO1 | 10 |
| **(OR)** | | | | |
| 8. | a. | Show that the ring of integers in an algebraic number field  is a Dedekind domain. | CO1 | 10 |
| b. | Let *A* be a Noetherian domain of dimension 1. Then prove that every non-zero ideal  in  can be uniquely expressed as a product of primary ideals whose radicals are all distinct. | CO1 | 10 |
|  | | **Compulsory**: |  |  |
| 9. | a. | State and Prove Nakayama’s lemma. | CO1 | 5 |
| b. | (i) Let  be prime ideals and let be an ideal contained in  Then  for some  (ii) Let  be ideals and let be a prime ideal containing Then  for some | CO1 | 15 |