Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_\_



**End Semester Examination – Nov / Dec – 2019**

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| **Code :** | **09MA314** | **Duration :** | **3hrs** |
| **Sub. Name :** | **THEORY OF NEAR-RINGS** | **Max. Marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | State and prove *Peirce Decomposition theorem* of a near-ring. | CO1 | 10 |
| b. | State and prove *first isomorphism theorem*. | CO1 | 10 |
| **(OR)** | | | | |
| 2. | a. | Let be faithful and assume that Then prove that  i. *Ώ = {0}* if and only if *Nc = {0}* if and only if *N ε η0.*  ii*. Ώ =*  if and only if *Nc =Mc (* if and only if  *Nc.* | CO1 | 15 |
| b. | Prove that *B* is a base for if and only if the nclusion map *i: B →*  can be extended to an *N*-isomorphism, where *Ø* is the free *N-*group on *Ø→Γ.* | CO1 | 5 |
|  |  |  |  |  |
| 3. | a. | Prove that for all , there exists such that N is embedded in | CO1 | 10 |
| b. | Let *N* be a near-ring. Then prove that  i. *N* is abelian and commutative if and only if *N* is commutative ring.  ii. *N* is abelian and distributive if and only if *N* is a ring.  iii. *N2=N* and *N* is distributive if and only if *N* is a ring. | CO1 | 10 |
| **(OR)** | | | | |
| 4. | a. | Prove that each of the following conditions imply to a near-ring N with identity to be abelian.  i. for all n ε N: n(-1) = -n  ii. N is finite and for all n ε N: n(-1)= n implies n = 0.  iii. for all n ε N, there exists hεN such that n = h+h and n(-1) = n implies n = 0. | CO1 | 10 |
| b. | Let *N* be a near-ring. Then prove that  i. *n ε N* is right cancellable if and only if *n* is not a right zero divisor.  ii. If *n ε N0* is left cancellable, then *n* is not a left zero divisor.  iii. If *N ε 0*, then the left cancellation law implies the right one. | CO1 | 10 |
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| 5. | a. | For each family *{Ik}kεK* of ideals of *N,* prove that the following conditions are equivalent:  i. The sum of the *Ik’s* is direct,  ii. The sum of the normal subgroups *(Ik , +)* is direct,  iii. For all *k* in *K: Ik= {0}.* | CO1 | 16 |
| b | If *N* is zero symmetric near-ring and is direct, then elements of different *Ik’s* have product *0*. | CO1 | 4 |
| **(OR)** | | | | |
| 6. | a. | If *I* and *J* are ideals of *N,* then prove that is an ideal of *J* and . | CO1 | 10 |
| b. | Define *distributive sums* and prove that if *N* is zero-symmetric near ring, the each direct sum of ideals is distributive. | CO1 | 10 |
|  |  |  |  |  |
| 7. | a. | Let *N* be a near-ring and *P* an ideal of *N.* Then prove that the following conditions are equivalent:  i. *P* is a prime ideal of *N,*  ii. For any ideals *I* and *J* of *N*, *(IJ)⊆ P* implies *I⊆P* or *J⊆P*,  iii. For any *i, j εN: iεP\I* and *jεP\J* imply *(i)(j)⊈ P,*  iv. For all ideals *I* and *J* of *N: I⊃P* and *J⊃P* imply *IJ⊈P,*  v. For all ideals *I* and *J* of *N: I⊈P* and *J⊈P* imply *IJ⊈P.* | CO1 | 15 |
| b. | If *I* is a direct summand and *P* is a prime ideal of *N,* then prove that *P∩ I* is a prime ideal of *N.* | CO1 | 5 |
| **(OR)** | | | | |
| 8. | a. | Let *I* be an ideal of *N.* Then prove that *N* is nilpotent if and only if *I* and *N/I* are nilpotent. | CO1 | 10 |
| b. | Let *N* be a zero symmetric near-ring with *DCCI.* Then prove that *N* is prime if and only if *N* has a smallest ideal *I* of *N* under all non-zero ideals and *I* is not nilpotent. | CO1 | 10 |
|  | | **Compulsory**: |  |  |
| 9. |  | Let *N* be *dg* by *D* and *Γ* be an *N-*group. Then prove that  i. If *Δ* is normal subgroup of *(Γ, +),* then *Δ ≤*  ii. *N2* is abelian if and only if *N* is distributive,  iii. *N* is abelian if and only if *N* is ring,  iv. If , then *N* is distributive if and only if *N* is abelian if and only if *N* is a ring. | CO1 | 20 |