Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_\_



**End Semester Examination – Nov / Dec – 2019**

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| **Code :** | **16AE2008** | **Duration :** | **3hrs** |
| **Sub. Name :** | **ADVANCED SPACE DYNAMICS** | **Max. Marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | Define a central orbit. If f is the law of attraction towards the origin, write the equation of motion. Prove that the angular momentum is a constant vector. Prove that the motion takes place in a plane which passes through the origin. | CO1 | 10 |
| b. | If two point masses m1and m2 are acted upon only by the mutual force of gravity between them, prove that the centre of mass of this system moves with constant velocity in a straight line. Prove that the motion of m2 around m1 is governed by a second-order non-linear differential equation. | CO1 | 10 |
| **(OR)** | | | | |
| 2. |  | Explain Lambert’s problem. Derive Lambert’s theorem analytically. | CO1 | 20 |
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| 3. |  | Derive the fifth-degree algebraic equations to find the locations of the collinear Lagrangian point L1, L2 and L3. | CO2 | 20 |
| **(OR)** | | | | |
| 4. |  | To study the motion near the equilibrium points, expand the force function Ω around a Lagrangian point. Find the linearized variational equation of motion in two dimensions.  Derive the fourth-degree characteristic equation  λ4 + (4 - Ωxx- Ωyy)λ2 + {ΩxxΩyy- (Ωxy)2}= 0,  at the Lagrangian points. | CO2 | 20 |
|  |  |  |  |  |
| 5. | a. | Find the second-order derivatives at the collinear points. Prove that the two roots of the characteristic equation are real and two roots are imaginary and the solution in general is unstable. | CO2 | 16 |
| b. | Prove that the location of the equilateral point L4 is given by  x = μ – 0.5 and y = 31/2/2, μ is the mass parameter. | CO2 | 4 |
| **(OR)** | | | | |
| 6. |  | Prove that the second-order derivatives at the equilateral point L4 are  Ωxx= 3/4, Ωxy = 3.31/2 (μ - 1/2)/2, Ωyy = 9/4.  Using these values of partial derivatives, prove that the characteristic equation is λ4 + λ2 + 27μ (1 - μ)/4 = 0.  Find the value of the critical mass μ0 = 0. 03852 at the equilateral equilibrium points. Prove that all the four roots of the characteristic equation are pure imaginary if 0 ≤μ<μ0. | CO2 | 20 |
|  |  |  |  |  |
| 7. | a. | By rotating the axes through an angle α at L4, prove that  tan 2 α = 31/2(1- 2 μ). | CO2 | 14 |
| b. | Construct the principal system of co-ordinates at the equilateral point L4 to show the angle α, which the major-axis of the elliptic orbit makes with the ξ-axis for mass ratio μ = 0.15 in the restricted three-body problem. | CO2 | 6 |
| **(OR)** | | | | |
| 8. | a. | Explain the halo orbits around the collinear points. | CO2 | 4 |
| b. | Define three-dimensional restricted three-body problem. | CO2 | 4 |
| c. | Prove Tisserand's criterion for the identification of comets. | CO2 | 12 |
|  | | **Compulsory**: |  |  |
| 9. | a. | Define the mass parameter μ. Derive the equations of motion for the planar restricted three body problem in synodic (rotating) coordinate system. Write the two equations to find the locations of the five equilibrium points. Derive Jacobi integral. | CO2 | 17 |
| b. | Find the mass parameter (μ) of Sun-Jupiter system. Masses of Sun and Jupiter are 1.989x1030 and 1.899x1027 kg, respectively. | CO2 | 3 |