Karunya University
(Karunya Institute of Technology and Sciences)
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End Semester Examinations - Nov-Dec 2015 Exams

14MA1001 Basic Mathematics for Engineering

Set A — **Time : 3 hrs**
Total Marks: 100

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1. (a) Resolve $\frac{x^2+x+1}{x^2-5x+6}$ into partial fractions. (10 marks)
   (b) Expand $(2x-3y)^6$ using binomial theorem. (10 marks)

OR

2. (a) If $A+B+C = 180^\circ$. Prove that $SinA + SinB + SinC = 4\,Cos\frac{A}{2}\;Cos\,\frac{B}{2}\;Cos\,\frac{C}{2}$ (10 marks)
   (b) Find the angle between the lines joining the points $(-1, 2)$,$(3, -5)$ and $(-2,3)$,$(5,0)$. (5 marks)
   (c) If A and B be the points $(1, 5)$ and $(-4, 7)$ then find the point P which divides AB internally in the ratio $2:3$ (5 marks)

3. (a) Find $\frac{dy}{dx}$ for $\frac{(1-x)\sqrt{x^2+2}}{(x+3)\sqrt{x-1}}$ (10 marks)
   (b) Find $\frac{dy}{dx}$ where $y = 3\,x^4 e^x + 2sinx + 7$ (5 marks)
   (c) Differentiate: $y = (x^2+2)(x^2-1)$ (5 marks)

OR

4. (a) Integrate $\int x\,tan^{-1}(x)\;dx$ (10 marks)
   (b) Integrate $\int \frac{1}{(x^2+1)(x+1)^2}\,dx$ (10 marks)

5. (a) Expand $x^2y + 3y - 2$ in powers of $x-1$ and $y+2$ up to the third term using Taylor Series. (10 marks)
   (b) If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ (10 marks)

OR

6. (a) Expand $\frac{x}{(x+1)(x-3)}$ about the point $x = 0$ using Taylor Series. (10 marks)
   (b) If $u = 2xy$, $v = x^2 - y^2$ and $x = r\,cos\theta$ and $y = r\,sin\theta$. Evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$ (10 marks)

7. (a) If $\vec{a} = \vec{i} + 2\vec{j} - \vec{k},\ \vec{b} = \overrightarrow{2i} - \vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{j} + \vec{k}, \vec{d} = \overrightarrow{3i} + \vec{j} - \overrightarrow{2k}$ then

   Find **(i)** $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ **(ii)** $\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$ **(iii)** $\vec{a}.\left(\vec{b}\times\vec{c}\right)$ **(iv)** $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)$ (10 marks)

   (b) Show that the following two lines are skew lines and hence find the distance between them $\vec{r} = (\vec{i}-\vec{j}) + t\left(\overrightarrow{2i}+\vec{k}\right)$ and $\vec{r} = \left(\overrightarrow{2i}-\vec{j}\right) + s\left(\vec{i}+\vec{j}-\vec{k}\right)$ (10 marks)

OR

8.

(a) Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect and hence find the point of intersection. (10 marks)

(b) Find the vector and Cartesian equation of the plane passing through the points $(1, -2, 3)$ and $(-1, 2, -1)$ and parallel to $2\vec{i}+3\vec{j}+4\vec{k}$ (10 marks)

9.

(a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (10 marks)

(b) Solve using Cramer's Method (10 marks)

$3x + y + 2z = 3$
$2x - 3y - z = -3$
$x + 2y + z = 4$

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End Semester Examinations - Nov-Dec 2015 Exams

14MA1002 Calculus and Statistics

Set A

Time : 3 hrs
Total Marks: 100

1.

a) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x} + \sin 2x$. (8)

b) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (12)

OR

2.

a) Find the complete solution of $(D^2 - 2D + 2)y = e^x \cos x$. (8)

b) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (12)

3.

a) Evaluate $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dydx$, by changing the order of integration. (12)

b) Evaluate $\int_0^a\int_0^b\int_0^c (x^2 + y^2 + z^2)dxdydz$. (8)

OR

4.

a) Using triple integration, find the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = a$ and $z = 0$. (10)

b) Evaluate $\iint r\sin\theta drd\theta$ over the cardioid $r = a(1 - \cos\theta)$ above the initial line. (10)

5.

a) State and prove the relation between beta and gamma functions. (10)

b) Show that $\int_0^{\pi/2}\sqrt{\sin\theta}d\theta . \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$. (10)

OR

6.

a) Show that $\beta(p,q)=\begin{cases}\int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}}dy \\ \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}dx\end{cases}$. (12)

b) Express $\int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$ in terms of gamma function. (8)

7.

a) Form the partial differential equation by eliminating the arbitrary function from $z = f(x+at)+g(x-at)$. (10)

b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. (10)

OR

8.

a) Solve $p^2+q^2=4pq$. (8)

b) Solve $(D^2-3DD'+2D'^2)z = e^{2x+3y}+\sin(x-2y)$. (12)

9.

a) Find the mean, median and standard deviation for the following data.

Weight (in gm)	0-10	10-20	20-30	30-40	40-50	50-60
No. of articles	14	17	22	26	23	18

(10)

b) Find the correlation coefficient between x and y from the following data.

x	78	89	97	69	59	79	68	57
y	125	137	156	112	107	138	123	108

(10)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2001 Vector Calculus and Complex Analysis

Set B **Time : 3 hrs**
Total Marks: 100

1.
a). Determine f(r) , so that the vector f(r) $\vec{r}$ is both solenoidal and irrotational. (12 marks)

b). If $\nabla\phi = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$, find φ. (8 marks)

OR

2.
a) Find the directional derivative of $x^2 + 2xy$ at (1,-1,3) in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$ (7 marks)

b) Find the velocity vector , the speed and the acceleration vector of the particle whose path is given by $\vec{r} = 3\cos 2t\,\vec{i} + 2\sin 3t\,\vec{k}$. (6 marks)

c) If $\vec{v} = \vec{w} X \vec{r}$, then prove that $\vec{w} = \frac{1}{2} curl\,\vec{v}$ where $\vec{w}$ is a constant vector.(7 marks)

3.
a) Verify Green's theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$. where C is the boundary of the region defined by x=0,y=0, x+y=1. (12 marks)

b) Verify Stoke's theorem for $\vec{F} = (2x - y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on z=0 plane.(8 marks)

OR

4.
a) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\,\vec{i} + (y^2 - zx)\,\vec{j} + (z^2 - xy)\,\vec{k}$ taken over the rectangular parallel piped enclosed by x=0, x=a, y=0,y=b,z=0,z=c. (15 marks)

b) If $\vec{F} = (3x^2 + 6y)\,\vec{i} - 14yz\,\vec{j} - 20xz^2\,\vec{k}$, then evaluate $\int_c \vec{F}.\vec{dr}$ from (0,0,0) to (1,1,1) along the curve x=t, y=t^2, $z = t^3$ (5 marks)

5.
a) If f(z) = u+iv is an analytic function where $u = x\sin x\cosh y - y\cos x\sinh y$, then find f(z) and v. (10 marks)

b) Prove that f(z) = $z\bar{z}$ is nowhere analytic .(4 marks)

c) If f(z) is analytic function, prove that

(i) $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\log|f(z)| = 0$ (ii) $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})amp f(z) = 0$ at the points where f(z)≠o (6 marks)

OR

6.

a) Show that w=logz is analytic in the complex plane except at the origin and its derivative is $\frac{1}{z}$. (10 marks).

b) prove that $\frac{d}{dz}(e^z) = e^z$ (7 marks)

c) Check whether $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic or not.(3 marks)

7.

a) Find the bilinear transformation which maps (-i, 0, i) onto (-1, i, 1) (10 marks)

b) Find the image of the z-plane bounded by the lines y=0, x=1, y=x by the transformation w=z^2 (10 marks)

OR

8.

a) Find the image of the strip 1<x<2 under the transformation w= $\frac{1}{z}$ (10 marks)

b) Discuss the transformations

(i) w= z+ $\frac{1}{z}$ (ii) w= coshz (10 marks)

9.

a). Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ (10 marks)

b) Evaluate $\oint_C \frac{z-3}{z^2+2z+5} dz$ where C: $|z+1-i| = 2$. (10 marks)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2002 Fourier Series and Applications

Set A — **Time : 3 hrs**
Total Marks: 100

1.

a) Write the Fourier series of the function f(x) in the interval (c, c+2π) (3)

b) Find the Fourier series of periodicity 2π for

$$f(x)= \begin{matrix} x, 0<x<\pi \\ 2\pi - x, \pi < x < 2\pi \end{matrix}$$ (17)

OR

2.

a) Express $f(x)=x^2, -\pi<x<\pi$, as a Fourier series of periodicity 2π. Hence deduce that

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+...=\frac{\pi^2}{6}$$
$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...=\frac{\pi^2}{12}$$
$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+....=\frac{\pi^2}{8}$$ (20)

3.

a) Define harmonic analysis. (3)

b) Compute the first two harmonics of the Fourier series for f(x) from the following data (17)

x	30	60	90	120	150	180	210	240	270	300	330	360
f(x)	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

OR

4.

a) Express $f(x)=x(\pi-x), 0<x<\pi$, as a Fourier series of periodicity 2π containing (i) sine terms only (ii) cosine terms only. Hence deduce

$$1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+...=\frac{\pi^2}{32}$$
$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...=\frac{\pi^2}{12}$$ (20)

5.

a) State two assumptions made in deriving one dimensional wave equation (4)

b) Derive D'Alembert's solution of wave equation. Also find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x)=a(x-x^2)$ (16)

OR

6. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating giving each point a velocity $3x(l - x)$, find the displacement. (20)

7. a) State any two empirical laws assumed to derive one dimensional wave equation (4)

b) A rod 30 cm. long, has its ends A and B kept at 20°C and 80°C., respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u(x,t) taking x = 0 at A. (16)

OR

8. a) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions:

$$(i) u(0,t) = 0, t \geq 0$$
$$(ii) u(\pi,t) = 0, t \geq 0$$
$$(iii) u(x,0) = \pi x - x^2, 0 < x < \pi$$

(20)

9. a) Write down the two dimensional heat equation in steady state (2)

b) A rectangular plate with insulated surfaces is 8c.m wide and so long compared to its width as an infinite plate. If the temperature along short edge $y = 0$ is $u(x,0) = 100 \sin\left(\frac{\pi x}{8}\right), 0 < x < 8$ while two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept 0°C. Find the steady-state temperature at any point of the plate. (18)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2003 Mathematical Transforms

Set B **Time : 3 hrs**
Total Marks: 100

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1. . (a) Find $L\left(\frac{1-\cos t}{t}\right)$ (10)

   (b) Find $L\left(e^{-4t} t \sin 3t\right)$ (10)

OR

2. (a) Evaluate $L\left(e^{-3t}\int_0^t \frac{\sin t}{t}\,dt\right)$ (10)

   (b) Find Laplace transform of the following periodic function (10)
   f(t) = k ; 0<t<a
   -k; a<t<2a

3. (a) Find $L^{-1}\left(\frac{4s+5}{(s+2)(s-1)^2}\right)$ (10)

   (b) Solve $y'' - 3y' + 2y = 4$ given y(0)=2, $y'(0) = 3$. (10)

OR

4. (a) Find $L^{-1}\left(\log\frac{s^2+1}{s(s+1)}\right)$ (10)

   (b) Using convolution find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ (10)

5. (a) Find Fourier transform of $f(x) = \begin{cases} a-|x| & ;|x|<a \\ 0 & ;|x|>a \end{cases}$

   and hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. (10)

   (b) Find finite Fourier sine and cosine transform of $f(x) = \begin{cases} 1 & 0<x<\frac{\pi}{2} \\ 0 & \frac{\pi}{2}<x<\pi \end{cases}$ (10)

OR

6. a. Find FFST and FFCT of $\left(\frac{x}{\pi}\right)$ in the interval $(0, \pi)$. (10)

   b. Find the Fourier sine transform of $e^{-|x|}$.

   Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx = \frac{\pi e^{-m}}{2}$, m>0.

   (10)

7. (a) Derive z(n), z(n^2) and hence find $z\left(\frac{(n+1)(n+2)}{2}\right)$ (10)

(b) Derive $z(\cos\omega t)$ and $z(\sin\omega t)$ (10)

OR

8. (a) Derive $z(n^2e^{an})$ (10)

(b) Derive z(t) and hence find $z(te^{-2t})$ (10)

9. (a) Using partial fraction find $Z^{-1}\left(\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right)$. (10)

(b) Solve $y(n+2)-4y(n+1)+4y(n)=0$ given that y(0)= 1 and y(1)=0 (10)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2004 Laplace Transforms, Fourier Series and Transforms

Set B | **Time : 3 hrs**
Total Marks: 100

1.

(a) Find $L(\sin^3 2t)$ **(10)**

(b) Find $L\left(\frac{\sin at}{t}\right)$ **(10)**

OR

2.

(a) Evaluate $L\left(t\int_0^t \frac{e^{-t}\sin t}{t}dt\right)$ **(10)**

(b) Find $L\left(t\int_0^t\left(\frac{e^{-t}-e^{-3t}}{t}\right)dt\right)$ **(10)**

3.

(a) Solve $y''+4y'+5y=0,\ y(0)=0,\ y'(0)=1$ **(10)**

(b) Find $L^{-1}\left(\log\left(\frac{s^2+1}{s(s+1)}\right)\right)$ **(10)**

OR

4.

(a) Solve $y''+y=\sin t,\ y(0)=1,\ y'(0)=\frac{1}{2}$ **(10)**

(b) Using convolution find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$ **(10)**

5.

(a) Find Fourier transform of $f(x)=\begin{cases}1 & ;|x|<1\\ 0 & |x|>1\end{cases}$.Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ **(10)**

(b) Find finite Fourier sine transform of $f(x)=\frac{x}{\pi}$ $(0,\pi)$ **(10)**

OR

6.

(a) Find Fourier transform of $f(x)=\begin{cases}a-|x| & ;|x|<a\\ 0 & |x|>a\end{cases}$.Hence evaluate $\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt$ **(10)**

(b) Find finite Fourier cosine transform of $f(x)=e^{ax}$ $(0,l)$ **(10)**

7. a). Find the Fourier series of the periodic function f (x) with period 2π given by

$f(x)=0$ in $-\pi\le x\le 0$, and x^2 in $0\le x\le\pi$. **(14)**

b) Find the half range cosine series of $f(x)=(\pi-x)^2$ in the interval $(0,\pi)$. **(6)**

OR

8. a) Obtain the Fourier series of $f(x)=x\cos x$ in $-\pi<x<\pi$, **(14)**
b) Express $f(x)=x$ in half range cosine series in the range $0<x<\ell$. **(6)**

9. a)Find the Fourier series up to second harmonic for the following data

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	0.8	0.6	0.4	0.7	0.9	1.1	0.8

(15)

b) Find the complex form of the Fourier series of $f(x) = e^x$ in $-\ell < x < \ell$. **(5)**

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2005 Mathematical Foundation

Set A

Time : 3 hrs
Total Marks: 100

1.

(a) Prove that $\cosh 5x = 16\cosh^5 x - 20\cosh^3 x + 5\cosh x$ (8)

(b) Prove that $\dfrac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$ (12)

OR

2.

(a) Find the real and imaginary part of $\sin(x+iy)$ (5)

(b) If $x+iy = \sin(A+iB)$ then prove that (i) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$

(ii) $\dfrac{x^2}{\cos^2 B} - \dfrac{y^2}{\sin^2 B} = 1$ (15)

3.

(a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (10)

(b) Determine whether the following set of equations are consistent or not. (6)

$x+y+z=2,\ 2x-y+z=5,\ 2x-z=0$

(c) Write any four properties of eigen values of a matrix. (4)

OR

4. State Cayley-Hamilton theorem and verify Cayley Hamilton's theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$. Hence find A^{-1}. (20)

5.

(a) If $y = x^4 e^x + 2\cos x + 7$ then find $\dfrac{dy}{dx}$. (6)

(b) If $y = \dfrac{2x-3}{4x+5}$ then find $\dfrac{dy}{dx}$. (7)

(c) If $y = (\sin x)^x$ then find $\dfrac{dy}{dx}$. (7)

OR

6.

(a) If $y = e^x \cos x$ then find $\frac{dy}{dx}$. (6)

(b) If $y = \sin(ax+b)^2$ then find $\frac{dy}{dx}$. (6)

(c) If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$ then find $\frac{dy}{dx}$. (8)

7.

(a) Evaluate $\int (x^3 + 4x^2 + 3x + 2)dx$ (4)

(b) Evaluate $\int \frac{(6x+5)dx}{\sqrt{3x^2+5x+6}}$ (6)

(c) Evaluate $\int_0^{\pi} x^4 \sin 2x dx$. (10)

OR

8.

(a) Evaluate $\int \frac{dx}{(x+1)(x+2)}$ (6)

(b) Evaluate $\int x^{16}(1+x^{17})^4 dx$ (6)

(c) Evaluate $\int_0^{\pi} x^2 \cos 3x dx$. (8)

9.

(a) Solve $(D^2 - 4D + 4)y = 8\sin 2x + e^{2x}$. (8)

(b) Solve $(D^2 + 2D + 1)y = 3e^x + \cos x + x^2$ (12)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2006 Numerical Mathematics and Computing

Set B — **Time : 3 hrs** — **Total Marks: 100**

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1. (a) Expand $\sqrt{1+h}$ in powers of h. Then compute $\sqrt{1.00001}$ and $\sqrt{0.99999}$. (8)
(b) Find $P(3)$ from the polynomial $P(x) = -2x^3 + 3x^4 + 6x^2 - 7x + 11$ by using nested multiplication. (4)
(c) Write Pseudo code of Bisection method. (8)

OR

2. (a) Find a root of the equation $f(x) = x^3 - x - 1$ using bisection method. (10)
(b) Convert $(26031)_8$ to binary, decimal and hexadecimal forms. (10)

3. (a) Find a positive root of the equation $f(x) = x^3 + x^2 - 1$ using Newton Raphson method. (10)
(b) If a = 0.1 and b = 1.0, how many steps of the bisection method are needed to determine the root with an error of at most $\frac{1}{2}\times 10^{-8}$? (5)
(c) Determine $f[x_0, x_1, x_2, x_3]$ from the following table: (5)

x	0	1	3	2
y	2	1	5	6

OR

4. (a) Using Newton's algorithm, derive the polynomial of least degree that assumes these values: (10)

x	0	1	-1	2	-2
y	-5	-3	-15	39	-9

(b) From the given data

x	0	1	2	4	6
y	1	9	23	93	259

(i) Construct the divided difference table.
(ii) Using Newton's interpolating polynomial find an approximate value of f(4.2). (10)

5. (a) Find a polynomial of least degree using Lagrange's interpolating polynomial method from the following table: (10)

x	0	1	3	2	5
Y	2	1	5	6	-183

(b) Use inverse Lagrange interpolating polynomial method to find x when y=25 from the below table: (10)

x	1	2	3	4
y	2	1	6	47

OR

6. (a) Compute $\int_0^4 2^x dx$ using Romberg's algorithm. (10)
(b) Write pseudo code of Trapezoid rule. (10)

7. (a) Evaluate (i) $\int_{-1}^{1} \frac{1}{1+x^2} dx$ (ii) $\int_{0}^{1} \frac{1}{1+x^2} dx$ using Gaussian Quadrature formula when n=2. (5)

(b) Compute $\int_{1}^{4} \frac{1}{1+x^2} dx$ using (i) Simpson's one third rule and (ii) Simpson's three eight rule when n=6. Also find the actual integration. (15)

OR

8. (a) Determine the Gaussian quadrature formula when the interval is [-2,2] and the nodes are -1,0 and 1. (10)

b) How many subintervals are needed to approximate $\int_{0}^{1} \sin\left(\frac{\pi x^2}{2}\right) dx$ using (i) Simpson's one third rule and (ii) Simpson's three eight rule with an error of magnitude less than 10^{-3}. (10)

9. a) Determine the parameters a,b,c,d,e,f,g,h so that $S(x)$ is a natural cubic spline,

where $S(x)=\begin{cases} ax^3+bx^2+cx+d, x\in[-1,0] \\ ex^3+fx^2+gx+h, x\in[0,1] \end{cases}$. (10)

b) Find a quadratic spline interpolant for the following data: (10)

x	-1	0	1/2	1	2	5/2
y	2	1	0	1	2	3

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2008 Probability and Statistics

Set A

Time : 3 hrs
Total Marks: 100

1. a. The scores of two players X and Y in 12 rounds are given below, who is better player and more consistent player

X	74	75	78	72	78	77	79	81	79	76	72	71
Y	87	84	80	88	89	85	86	82	82	79	86	80

b. Find Quartile Deviation from the following data.

X	50	51	52	53	54	55	56
F	10	12	15	10	14	18	6

OR

2. a. Find Correlation coefficient between X and Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

b. Find the two Regression Lines from the following data.

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

3. a. One shot is fired from each of three guns. The chance to hit the target are 0.5, 0.6, 0.7respectively. If the events are independent, what is the probability that (i) exactly one hit is registered.(ii)atleast two hits are registered.(iii) no hits are registered

b. Box I contains 2 red, 3white,3 blue balls. Box II contains 4red, 1white, 3blue balls. Box III contains 3red, 4white, 5blue balls. A box is selected at random from which a ball is selected at random from which a ball is selected at random and it is observed to be red, what is the probability that box III was selected.

OR

4. a. The joint probability mass function of (X,Y) is p(x,y) = k(2x+3y) where x=0,1,2 ; y=1,2,3. Find the value of k.Find the marginal and conditional probability distributions. Also find the probability distribution of X+Y. (15 marks)

b. From the following probability distribution: (5 marks)

x	0	1	2	3	4
P(x)	k	3k	5k	2k	k

Find the value of k.

Find P(X<2).

Find mean of X.

5. a. Fit a binomial distribution to the following data and find theoretical frequencies.

X	0	1	2	3	4
Y	5	29	36	25	5

b. In a test of 2000 electric lamps it was found that the life of a particular brand was normally distributed with average life of 2040 hours and standard deviation of 60 hours. Estimate the number of lamps likely to burn for (i) more than 2150 hours (ii) less than 1950 hours (iii) more than 1920 hours but less than 2160 hours.

OR

6. a. In a certain factory producing razor blades there is a small chance of 1/500 for any blade to be defective. The blades are in packets of 10. Use poisson distribution to calculate the approximate number of packets containing (i) no defectives (ii) atleast two defectives (iii) atmost three defectives in a consignment of 10,000 packets.

b. A machine manufacturing screws is known to produce 5% defectives. In a random sample of 15 screws, find the probability using binomial distribution that there are (i) exactly three defectives. (ii) not more than three defectives (iii) no defectives.

7. a. In a random sample of 1000 persons from the city of Coimbatore, 400 were consumers of wheat. In a sample of 800 from the city of Madurai, 400 were consumers of wheat. Do these data, reveal a significant difference between the two cities so far as the proportion of wheat consumers is concerned.

b. The heights of college students in a city are normally distributed with standard deviation 6cms. The mean height of a sample of 100 students is 158cms. Test whether the mean height of college students in the city is 160cms.

OR

8. a. For a random sample of 10 persons treated with medicine A , the increase in weight in a certain period were 10,6,16,17,13,12,8,14,15,9. For another sample of 12 persons treated with medicine B, the increase in weight in the same period were 7,13,22,15,12,14,18,8,21,23,10,17. Check whether the estimates of population variance are significantly different.

b. The number of automobile accidents per week in a certain community are 12,8,20,2,14,10,15,6,9,4. Are the frequencies in agreement with the belief that accident conditions were the same during this 10weeks period.

9. Analyse the variance from the following latin square design and give your conclusion.

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2009 Statistical Data Analysis and Reliability Engineering

Set B

Time : 3 hrs
Total Marks: 100

1. a. Find Rank Correlation coefficient from the following data.

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

b. Find Correlation coefficient between X and Y.

X	45	56	39	54	45	40	56	60	30	36
Y	40	36	30	44	36	32	45	42	20	36

OR

2. Fit a straight line and a parabola to the following data.

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

3. a. Samples of two types of electric lights were tested for length of life and the following data were obtained.

Sample size	Mean	standard deviation
8	1234hrs	36hours
7	1036hrs	40hours

Test the hypothesis that TypeI is superior to Type II.

b. Given the following contingency table for hair colour and eye colour. Test whether hair colour and Eye colour are independent.

EYE COLOUR	HAIR COLOUR		
	FAIR	BROWN	BLACK
BLUE	15	5	20
GREY	20	10	20
BROWN	25	15	20

OR

4. a. For a random sample of 10 horses fed on diet A, the increase in weight in a certain period were 10,6,16,17,13,12,8,14,15,9. For another sample of 12 horses fed on diet B, the increase in weight in the same period were 7,13,22,15,12,14,18,8,21,23,10,17. Check whether the variance do not differ significantly.

b. The number of automobile accidents per week in a certain community are 12,8,20,2,14,10,15,6,9,4. Are the frequencies in agreement with the belief that accident conditions were the same during this 10weeks period.

5. From the following data, Analyse the variance and test whether there is difference between

i. Treatments ii.Plot of land

PLOTS OF LAND	TREATMENTS			
	A	B	C	D
I	38	40	41	39
II	45	42	49	36
III	40	38	42	42

OR

6. Analyse the variance from the following latin square design and give your conclusion.

D2	A1	C3	B2
B4	C3	A2	D5
A0	B1	D0	C1
C2	D3	B1	A2

7. From the following data , draw control charts of Mean and Range. Comment on state of control.

Sample number	1	2	3	4	5	6	7	8	9	10
Mean	62.5	57.7	60.2	66.2	63.5	67.7	63.7	64.7	58.2	67.5
Range	25	31	19	24	20	49	17	14	16	11

OR

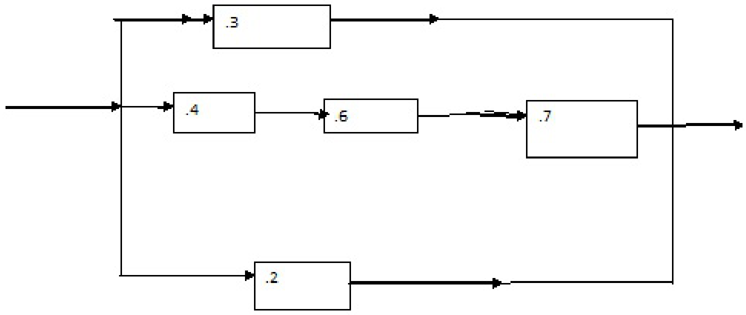
8. a. Explain Six sigma quality level with example in food industry. (5 marks)

b. A plant produces paper for newsprint and rolls of paper are inspected for defects. The results are tabulated, draw C-Chart and comment on state of control. (15marks)

Roll number	1	2	3	4	5	6	7	8	9	10	11	12	12	14	15
Numer of defects	2	4	3	1	1	2	5	3	6	7	3	1	4	2	1

9. a. Explain (i) High level Redundancy (ii) Low level Redundancy (8 marks)

b. Find Reliability of the system whose block diagram is given below. (12 marks)



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End Semester Examinations - Nov-Dec 2015 Exams

14MA2010 Discrete Mathematics

Set B | **Time : 3 hrs**
Total Marks: 100

1. (a) A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games and 50 do not watch any of the 3 kinds of games.
   (i) How many people in the survey watch all the 3 kinds of games?
   (ii) How many people watch exactly one of the sports? (10)

   (b). Using Euclidean Algorithm, find the greatest common divisor and least common multiple of the integers *1819* and *3587*. (5)

   (c) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent. (5)

OR

2. (a) Using Mathematical Induction, prove that $1+2+3+\ldots+n = \frac{n(n+1)}{2}$ *for* $n \geq 1$. (10)

   (b) Solve $d_n = 2d_{n-1} - d_{n-2}$ with initial conditions $d_1 = 1.5$ and $d_2 = 3$. (10)

3. (a) Let A = {1, 2, 3, 4, 5}and the relation R is defined by aRb iff a $\leq$ b. Find R, domain, range, matrix representation, digraph, indgrees, outdegrees of the relation R. (10)

   (b) For any integers a, b and m, prove that $a \equiv b(\text{mod}\, m)$ is an equivalence relation. (10)

OR

4. (a) Let $A = \{1,2,3,4\}$. Define the relation R on A, whose matrix is $M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   Find (i) reflexive closure (ii) symmetric closure (iii) transitive closure using Warshall's algorithm. (12)

   (b) Let A = {1, 2, 3, 4, 5}. Let M_R and M_S be the matrices of the relations R and S on A given below. Compute (i) $M_{\bar{S}}$, (ii) $M_{R^{-1}}$, (iii) $M_{R \cup S}$, (iv) $M_{R \cap S}$ and (v) $M_{S \circ R}$.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}, M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$ (8)

5. (a) Prove that $(Z^{+}, /)$ is a partially ordered set. (8)

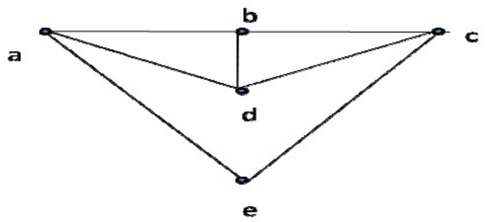
(b) Draw the Hasse Diagram of $(P(X), \subseteq)$, where X = {a, b, c}. (12)

OR

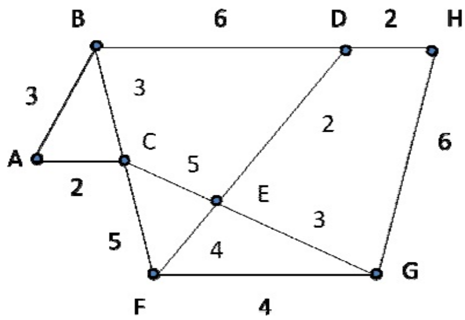
6. (a) Determine whether (D_{30}, /) is a lattice. Also find complements of each element. (14)

(b) Construct the truth table and draw the logic diagram for the Boolean polynomial
$p(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$ (6)

7. (a) Construct a spanning tree for the connected graph given below. Use 'c' as root. (10)

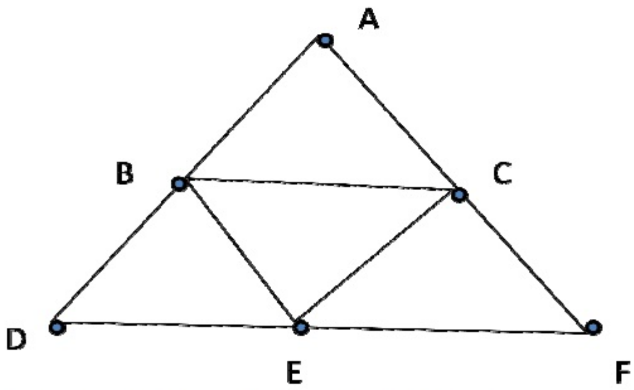


(b) Using Prim's algorithm, find the minimal spanning tree for the graph given below (10)

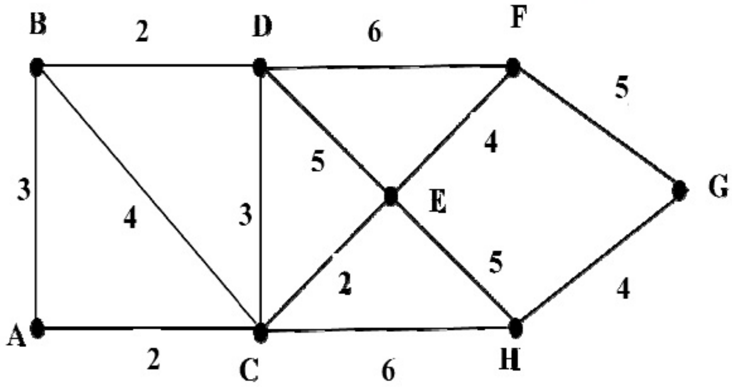


OR

8. (a) Use Fleury's Algorithm to find an Euler circuit for the graph given below.

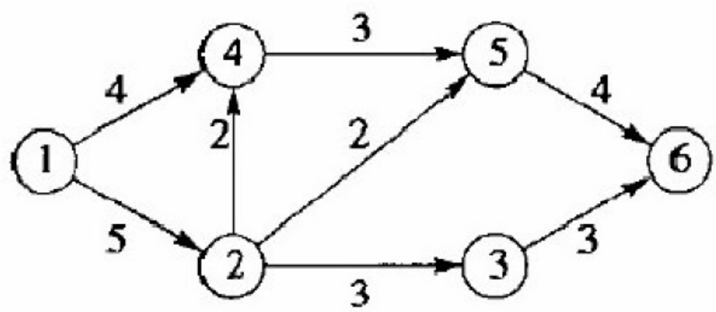


(b) Find a minimum Hamiltonian circuit for the graph given below. (4)



(c) Construct the tree of algebraic expression $(7+(6-2)) - (x - (y-4))$. (4)

9. (a) Find a maximal flow in the network given below by using labeling algorithm (14)



b. *Let m=2 and n=5,* let (6)

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Determine the group code function $e_H: B^2 \to B^5$

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2012 Numerical Methods

Set B **Time : 3 hrs**
Total Marks: 100

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1. a) Fit a straight line to the data given below. Also estimate the value of y at x=2.5 (10)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

b) Fit a second degree parabola to the following data, taking y as dependent variable. (10)

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

OR

2. a) From the table given below, find the best values of a and b in the law $y = ae^{bx}$ By the method of least squares. (10)

x	0	5	8	12	20
y	3.0	1.5	1.0	0.55	0.18

b) It is known that the curve $y = ax^b$ fits in the data given below. Find the best values Of a and b. (10)

x	1	2	3	4	5	6
f(x)	1200	900	600	200	110	50

3. a) Using Newton's Method , find the root between 0 and 1 of $x^3 = 6x - 4$ correct to 5 decimal places. (10)

b) Solve, by Gauss- Seidel method, the following systems:
28x+4y-z =32, x+3y+10z=24, 2x+17y+4z=35 (10)

OR

4. a) Solve the system by Gauss Elimination method
2x+3y-z =5,4x+4y-3z =3,2x-3y+2z =2 (10)

b) Solve the following system of equations by Gauss Jacobi method correct to 3 decimal Places: x+y+54z=110, 27x+6y-z=85, 6x+15y+2z=72 (10)

5. a) Using Lagrange's Interpolation Formula, find y(10) from the following table (10)

x	5	6	9	11
y	12	13	14	16

b) The population of a town is as follows. Using Newton's forward method (10)

Year (x)	1941	1951	1961	1971	1981	1991
Population-lakhs(y)	20	24	29	36	46	51

Estimate the population increase during the period 1946

OR

6. a) From the data given below, find the value of x when y=13.5 using Inverse Interpolation. (10)

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

b) Using Gauss's backward interpolation formula find the population for the year 1936 given that (10)

Year :(x)	1901	1911	1921	1931	1941	1951
Population in thousand(y)	12	15	20	27	39	52

7. a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson's rule (10)

b) Given $y' = -y$ and y(0)=1, determine the values of y at x=0.01, 0.02, 0.03, 0.04 by Euler method (10)

OR

8. a) Solve $y' = x + y$, given y (1) =0, and get y (1.1), y (1.2) by Taylor series method. (10)

b) Solve the equation $y' = 1 - y$ given y(0)=0 using Modified Euler's Method and tabulate the solutions at x=0.1, 0.2 (10)

9. a) Obtain the values of y at x=0.1,0.2 using Range- Kutta method of third order For the differential equation $y' = -y$, given y (0) =1. (10)

b) Find y (0.2) given $y' = y - x$,y (0)=2 taking h=0.1 using fourth order Range-kutta method. (10)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA2015 Probability, Random Process and Numerical Methods

Set A

Time : 3 hrs
Total Marks: 100

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1. (a) In a shooting test, the probability of hitting the target is $\frac{1}{2}$ for A ,$\frac{2}{3}$ for B and $\frac{3}{4}$ for C. If all of them fire at the target, find the probability that (i) None of them hit the target (ii) None of them hit the target. (10 marks)

(b) A and B alternately throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is $\frac{30}{61}$. (10 marks).

OR

2. (a) A bolt is manufactured by 3 machines A, B and C. A turns out twice as many items as B and machine B and C produce equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into one stock pile and 1 is chosen from this pile. What is the probability that it is defective? Assuming that the chosen bolt is defective, what is the probability that it is produced by machine B (10 marks)

(b) If A and B are two independent events, prove that (i) $\bar{A}$ and $\bar{B}$ are independent (ii) $\bar{A}$ and B are independent. (10 marks)

3. (a) If a random variable X has the following probability distribution (10 marks)

x	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	2k	0.3	3k

Find (i) the value of k (ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$
(iii) Evaluate the mean, variance of X

(b) The joint pdf of the RV (X, Y) is given by $f(x, y) = kxy\, e^{-(x^2+y^2)}$, $x, y > 0$
Find (i) the value of k (ii) the marginal and conditional distributions (iii) Prove that X and Y are independent. (10 marks)

OR

4. (a) A continuous RV X that can assume any value between x = 2 and x = 5 has a density function given by $f(x) = k(1 + x)$. Find (i) the value of k (ii) $P(X < 4)$ (iii) mean and variance. (10 marks)

(b) For the bivariate probability distribution of (X, Y) given below (10 marks)

	Y					
X	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

Find (i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv) $P(X \leq 1 / Y \leq 3)$
(v) $P(Y \leq 3 / X \leq 1)$ (vi) $P(X + Y \leq 4)$

5. (a) Fit a binomial distribution to the given data and calculate the expected frequencies (10 marks)

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

(b) In a certain factory turning razor blades, there is a small probability of $\frac{1}{500}$ of any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) No defective (ii) One defective (iii) Two defective (10 marks)

OR

6. (a) 1000 light bulbs with the mean life of 120 days are installed in a new factory, their length of life is normally distributed with standard deviation 20 days. How many bulbs will expire in (i) less than 90 days (ii) more than 90 days (iii) between 75 and 90 days? (10 marks)

(b) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the (i) probability that the repair time exceeds 2 hours. (ii) Conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours. (10 marks)

7. (a) Show that the random process $X\{t\} = A\ Cos\ (\omega t + \theta)$ is wide-sense stationary, if A and ω are constants and θ is a uniformly distributed RV in $(0,2\pi)$. (10 marks)

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using trapezoidal rule with h = 0.2. Hence obtain an approximate value of π . (10 marks)

OR

8. (a) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\ Cos(100\ t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ If tis correlation ergodic. (10 marks)

(b) The table below gives the velocity v of a moving particle at time t seconds. Using Simpson's Rule find the distance covered by the particle in 12 seconds and also the acceleration at t = 12 seconds also the acceleration at t = 12 seconds (10 marks)

t	0	2	4	6	8	10	12
v	4	6	16	34	60	94	136

9. (a) Apply the fourth order Runge-Kutta method to find y (0. 1) and y (0. 2) given that $y' = x + y$ and y (0) = 1. (14 marks)

(b) Given $y' = -y$ and y (0) = 1, determine the values of y at x = 0.01 (0.01) 0.04 by Euler Method (6 marks)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3003 Foundations of Mathematics and Statistics

Set B **Time : 3 hrs**
Total Marks: 100

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1. a) Find the greatest term in the expansion of $(1+x)^{\frac{13}{2}}$ when $x=\frac{2}{3}$. (5)

b) Find the middle term of $\left(\frac{y\sqrt{x}}{5}-\frac{5}{x\sqrt{y}}\right)^{12}$. (5)

c) Find the sum to infinity of the series $1+\frac{3}{4}+\left(\frac{3}{4}\right)\left(\frac{5}{8}\right)+\left(\frac{3}{4}\right)\left(\frac{5}{8}\right)\left(\frac{7}{12}\right)+...$ (10)

OR

2. a) Sum the series $\frac{1}{1!}+\frac{1+5}{2!}+\frac{1+5+5^2}{3!}+...$ (10)

b) Find the coefficient of x^n of $\frac{1+2x-3x^2}{e^x}$ (10)

3. a) Evaluate (i) $\underset{x\to 0}{Lt}\frac{(1+x)^3-1-3x}{(1+x)^2-1-2x}$ (ii) $\underset{x\to 2}{Lt}\frac{x^2-5x+6}{x^2-4}$ (5)

b) Investigate the continuity of $f(x)$ at c from $f(x)=\frac{\sin(x-c)}{x-c}$, if $x\neq c$ (5)
$=0$, if $x=c$

c) Derive the differential coefficient of $\sin x$. (10)

OR

4. a) Derive the product rule of differentiation. Also find the value of $\frac{d}{dx}\{xe^x\sin x\}$. (10)

b) A window has the form of a rectangle surmounted by a semicircle. If the perimeter is 40 ft., find its dimensions so that the greatest amount of light may be admitted. (10)

5. a) Evaluate (i) $\int\frac{\cos^2 x}{1-\sin x}dx$ (ii) $\int\left(\frac{3x^2+4x-5}{\sqrt{x}}\right)dx$ (iii) $\int\frac{(x+1)^4}{x^2}dx$ (15)

b) Evaluate $\int_0^{\frac{\pi}{6}}\cos^2\left(\frac{x}{2}\right)dx$. (5)

OR

6. a) Derive the integration by parts formula. Using the integration by parts, evaluate $\int x^2e^{3x}dx$. (10)

b) Evaluate using Bernoulli's formula (i) $\int x^5e^{-4x}dx$ (ii) $\int x^3e^{2x}dx$ (10)

7. a) A committee consists of 9 students, 2 of which are from first year, 3 from second year and 4 from third year. 3 students are to be removed at random. What is the chance that (i) 3 students belong to different classes (ii) 2 belong to the same class and third belong to the different class (iii) 3 belong to the same class (10)

b) A certain screw making machine produces on average of 2 defective screws out of 100 and packs them in boxes of 500. Find the probability that a box contains 15 defective screws. (5)

c) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. (5)

OR

8. a) Fit a Poisson distribution to the set of observations : (10)

x	0	1	2	3	4
f	122	60	15	2	1

b) Fit a Binomial distribution to the following data: (10)

x	0	1	2	3	4	5	6	7	8	9	10
f	6	20	28	12	8	6	0	0	0	0	0

9. a) A sample size of 600 persons selected at random from a large city shows that the percentage of males in the sample is 53. It is believed that the ratio of males to the total population in the city is $\frac{1}{2}$. Test whether the belief is confirmed by the observation. (5)

b) A test was given to a large group of boys who scored on the average 64.5 marks. The same test was given to a group of 400 boys who scored on average of 62.5 marks with a S.D of 12.5 marks. Examine the difference is significant. (5)

c) Two independent samples of sizes 7 and 6 have the following values: (10)

Sample A	28	30	32	33	29	34	33
Sample B	29	30	30	24	29	28	

Examine whether the samples have been drawn from the same population.

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3004 Advanced Calculus and Numerical Methods

Set A **Time : 3 hrs**
Total Marks: 100

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1. a) A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is set vibrating by giving each point of its velocity λ x(l-x), find the displacement of the string at any distance x from one end at any time t. (10)

   b) A rod of length 10cm has its ends A and B kept at 30°C and 100°C respectively until steady state conditions prevail. After some time temperature at A is reduced to 20°C and B is reduced to 40°C. Find the subsequent temperature distribution. (10)

OR

2. a) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

   (i) u is finite as $t \to \infty$.

   $(ii) \frac{\partial u}{\partial x} \to 0$ as x=0 and x=l.

   (iii) u= lx-x² for t=0, 0<x<l. (10)

   b) A tightly stretched string with fixed end points is initially in a position given by $y(x,0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement y(x,t). (10)

3. a) In a semicircular plate of radius 'a' cm has insulated faces and heat flows in plane curves. The temperature on the boundary is 0°C and the circumference is kept at 100°C. Find the steady state temperature distribution in the plate. (10)

   b) Find the extremal for the functional I(y) = $\int_0^{\frac{\pi}{2}} (y^2 - y'^2 - 2y \sin x)dx$,

   where y(0)=0, y($\frac{\pi}{2}$)=1. (10)

OR

4. . a) A rectangular plate with insulted surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along the short edge y=0 is given by

   $u(x,0) = 100 \sin \frac{\pi x}{8}, 0 < x < 8,$

   While the two long edges x=0 and x=8 as well as the other short edge are kept at 0°C. Find the Steady state temperature distribution u(x,y) in the plate. (10)

   b) Find the extremal for the functional I(y) = $\int_0^{\frac{\pi}{4}} (y''^2 - y^2 + x^2)dx$, where y(0)=0, y($\frac{\pi}{4}$)=$\frac{1}{\sqrt{2}}$, y'(0)=0,

   y'($\frac{\pi}{4}$)= $\frac{1}{\sqrt{2}}$. (10)

5. Find all the Eigen values and Eigen Vectors of the Matrix A = $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{pmatrix}$ by using Given's method. (20)

OR

6\. . a) Using Jacobi's method find all the Eigen values and Eigen vectors of the matrix

A = $\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$. (15)

b) Find the Numerically largest Eigen value and Eigen vector of the matrix A= $\begin{pmatrix} 3 & -5 \\ -2 & 4 \end{pmatrix}$ by using power method. (5)

7\. . a) Solve the equation $y^{II}+y+x=0$, $y(0)=y(1)=0$, using collocation method. (12)

b) By dividing the range into 10 equal parts find $\int_0^{\pi} \sin x\,dx$, by using Simpson's rule. Check the result by direct Integration. (8)

OR

8\. . a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg Method. Hence obtain an value for π. (10)

b) Evaluate $\int_1^{1.4}\int_2^{2.4} \frac{1}{xy}\,dx\,dy$, using Simpson's rule and hence check the result by direct Integration. (10)

9\. a) Obtain the cubic spline approximation for the function y=f(x), from the following data, given that $y_0^{I} = y_3^{I} = 0$. (15)

x	1	2	3	4
y	1	5	11	8

Hence find the values of y(1.5) and $y^{I}(2)$.

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using two point Gaussian Quadrature formula. (5)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3005 Calculus of Variations and Vector Spaces

Set B | **Time : 3 hrs**
Total Marks: 100

1. a. A curve C joining the points (x_1,y_1) and (x_2,y_2) is revolved about the X-axis. Find the shape of the curve so that the surface thus generated is a minimum. (10 Marks)

b. Find the extremal of the functional $\int_{0}^{\pi/2}(y'^2+z'^2+2yz)dx$ given that $y(o)=0, y(\pi/2)=-1, z(0)=0, z(\pi/2)=1$. (10 Marks)

OR

2. a. Determine the extremal of the functional $V(y(x))=\int_0^1(1+y''^2)dx$ that satisfies the conditions $y(0) = 1$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$. (10 Marks)

b. Find the plane curve of fixed perimeter and maximum area. (10 Marks)

3. a. Show that the integral equation $y(x)=\int_0^x(x+t)y(t)dt+1$ is equivalent to the differential equation $y''(x)-2xy'(x)-3y(x) = 0$ and the initial conditions $y(0) = 1$, $y'(0) = 0$. (10 Marks)

b. Transform the boundary value problem $y''+xy = 1$, $y(0) = y(1) = 0$ to an integral equation, finding the corresponding Green's function. (10 Marks)

OR

4. a. Find the characteristic numbers and eigen functions for the integral equation $y(x)=\lambda\int_0^{\pi}\cos(x+t)y(t)dt$ with degenerate kernels. (10 Marks)

b. Using the method of successive approximations, solve the integral $y(x)=x-\int_0^x(x-t)y(t)dt.$ (10 Marks)

5.

a. If $f: Z \to N$ is defined by $f(x) = \begin{cases} 2x-1; x>0 \\ -2x; x \le 0 \end{cases}$, then (i) prove that f is one - to - one and onto and (ii) determine f^{-1}. (6 Marks)

b. Define equivalence relation. If R is the relation on the set of positive integers such that $(a,b) \in R$ if and only if $a^2 + b$ is even, prove that R is an equivalence relation. (10 Marks)

c. Verify whether $w = \{\alpha = (a_1, a_2, a_3, ..., a_n) \varepsilon R^n \mid a_2 = 3a_1^{\ 2}\}$ is a subspace or not. (4 Marks)

OR

6. Prove that set of all polynomials of degree $\le$ n is a vector space V over a field F. (20 Marks)

7. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite dimensional and $\dim(W_1) + \dim(W_2) = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$. (20 Marks)

OR

8. a. Apply the Gram-Schmidt process to the vectors $\beta_1 = (3,0,4)$, $\beta_2 = (-1,0,7)$, $\beta_3 = (2,9,11)$ to obtain an orthonormal basis of R^3 with the standard inner product. (10 Marks)

b. Prove that the standard vectors (1,0,0), (0,1,0) and (0,0,1) can be written as a linear combination of α_1, α_2, α_3 where $\alpha_1 = (1,0,-1)$, $\alpha_2 = (1,2,1)$, $\alpha_3 = (0,-3,2)$ forms a basis of R^3. (10 Marks)

9.

a. Find the inverse Z-transform of $\dfrac{2z^2+3z}{(z+2)(z-4)}$. (10 Marks)

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0$, $y_1 = 0$ using Z-transform. (10 Marks)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3006 Graph Theory and Random Process

Set A — **Time : 3 hrs** — **Total Marks: 100**

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1. a. Let the number of edges in a graph G be m. Prove that G has Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$. (10)

   b. Define Discrete graph, Complete Graph, Linear Graph. Give example for each graph. (5)

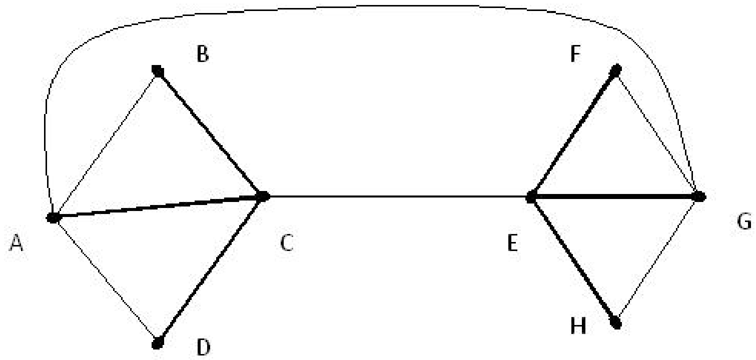
   c. State and prove max flow and min cut theorem (5)

OR

2. a. Let G_e be the subgraph of G obtained be deleting the edge e and let G^e be the quotient graph of G obtained by merging the end points of e. Prove that $P_G(x) = P_{G_e}(x) - P_{G^e}(x)$. (8)

   b. Define Chromatic number and Chromatic Polymomial with example. (4)

   c.. State Fleury's algorithm and construct an Euler circuit for the graph shown below (8)



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3. a. Let $(T; v_0)$ be a rooted tree then prove the following: (8)
   1. There are no cycles in T.
   2. v_0 is the only root of T.
   3. Each vertex in T, other than v_0, has in-degree one and v_0 has in-degree zero.

   b. Construct the tree of the algebraic expression $(x \div y)-((x \times 3)(z \div 4))$ and perform the postorder, preorder and inorder search and find the respective notations. (7)

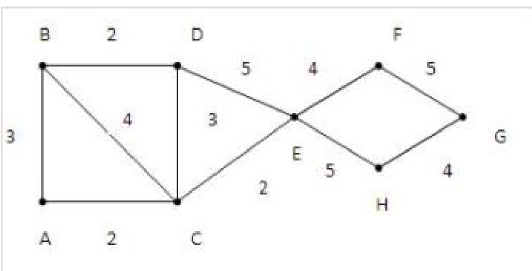
   c Evaluate the following expression which is given in polish and reverse polish notation. (5)

   1. $\times$ - + 34 - 72 $\div$ 12 $\times$ 3 - 64
   2. 3 7 $\times$ 4 - 9 $\times$ 6 5 $\times$ 2 + $\div$

OR

4. a. Prove that a tree with n vertices has n - 1 edges. (8)

   b. State Kruskal's Algorithm and use it to find a minimal spanning tree for the following the graph given below (12)



5. a. The chances of A, B and C becoming the general manager of a certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B and C become general manager are 0.3, 0.7 and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been appointed as general manager? (10)

b. The probability function of an infinite discrete distribution is given by $P(X = j) = 1/2^j$, $j = 1, 2, \ldots$ Verify that the total probability is 1 and the mean and variance of the distribution. Find also P(X is even) and P(X is divisible by 3). (10)

OR

6.

a. There are three boxes containing respectively 1 white, 2 red, 3 black balls; 2 white, 3 red and 1 black balls; 3 white, 1 red, 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from second box? (10)

b. The probability mass function of a RV X is defined as $P(X=0) = 3c^2$, $P(X=1) = 4c-10c^2$, and $P(X=2) = 5c - 1$ where $c>0$, find the following (i) the value of c, (ii) $P(0 < X < 2 / X > 0)$, (iii) the cdf of X, (iv) Mean (v) Variance. (10)

7. a. Two random processes X(t) and Y(t) are de ned by $X(t) = A\cos w_0 t + B\sin w_0 t$ and $Y(t) = B\cos w_0 t - A\sin w_0 t$. Show that X(t) and Y(t) are jointly wide sense stationary if A and B are uncorrelated RVs with zero means and the same varaiances and w_0 is a constant. (15)

b. Prove that the Auto correlation Function of a stationary process is an even function. (5)

OR

8. a. State and prove mean Ergodic theorem. (10)

b. If the WSS process $\{X(t)\}$ is given by $X(t) = A\cos(\lambda t+\theta)$, where θ is uniformly distributed over $(-\pi, \pi)$. Prove that $\{X(t)\}$ is correlation ergodic. (10)

9. a. Customer arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cards. Other can wait outside this space. (i) what is the probability that an arriving customer can drive directly to the space in front of the window? (ii) What is the probability that an arriving customer will have to wait outside the indicated space? (iii) How long the arriving customer is expected to wait before starting service?

(10)

b. A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The ength of service on these calls is approximately exponentially distributed with mean length 5 minutes. (i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day? (ii) If the subscribers will wait and are serviced in turn, what is the expected waiting time? (10)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3008 Computational Mathematics

Set B | **Time : 3 hrs**
Total Marks: 100

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1. (a). Find the extremal for

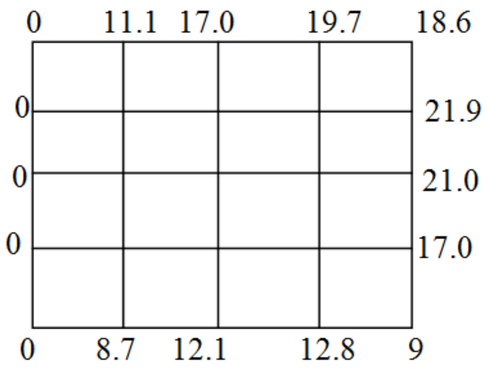
$$I=\int_0^{\frac{\pi}{4}}(y''^2-y^2+x^2)dx,\ y(0)=0,\ y'(0)=1,\ y\left(\frac{\pi}{4}\right)=y'\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}. \qquad (10)$$

(b). Prove that the shortest distance between two points in a plane is a straight line. (10)

OR

2. (a). Show that the functional $\int_0^{\frac{\pi}{2}}\left[2xy+\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]dt$ such that x(0) = 0, $x\left(\frac{\pi}{2}\right)=-1$, y(0) = 0 and $y\left(\frac{\pi}{2}\right)=1$, is stationary for $x=-\sin t$ and $y=\sin t$ (20)

3. (a). Solve the elliptic equation $u_{xx}+u_{yy}=0$ for the following square mess. (14)



(b).Classify the following partial differential equation

(i) $xf_{xx}+yf_{yy}=0$ (6)

(ii) $f_{xx}-2f_{xy}+4f_{yy}=0$

OR

4. (a). Solve the steady state two dimensional heat equation $u_{xx}+u_{yy}=-10(x^2+y^2+10)$ over the square mesh of sides x = 0 = y, x = 3 = y with u = 0 on the boundary and mesh length 1 unit. (10)

(b). Solve the one dimensional heat equation $u_{xx}=u_t$ subject to the conditions u(x, 0) = 0, u (0, t) = 0 and u (1, t) = t using Crank – Nicolson method for two steps in t direction with h=1/4. (10)

5. (a). Solve $\frac{dy}{dx} = 1 - y$ and y(0) = 0, at x=0.1,0.2 ,using (10)

(i). Euler method.

(ii). Modified Euler method

(iii). Compare the result with exact solution

(b). Find the dominant eigenvalue and the corresponding eigenvector using Power method $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ (10)

OR

6. (a).Solve $\frac{dy}{dx} = x + y$, given y(0) =1. Obtain the values of y (0.1), y (0.2) using Picards method and check your answer with the exact solution. (10)

(b). Solve y”-y +x =0 with y(0)=0, y(1)=0 using Raleigh-Ritz method. (10)

7. (a). Solve the equations $x^2 + y^2 = 16,\ x^2 - y^2 = 4$, $x_0 = 2\sqrt{2},\ y_0 = 2\sqrt{2}$ by Newton-Raphson method (2 iterations). (10)

(b). Solve by Gauss elimination and Gauss Jordan method

$2x + 3y - z = 5,\ \ 4x + 4y - 3z = 3,\ \ 2x - 3y + 2z = 2$ (10)

OR

8. (a). Find the positive root of $x^3 - 4x + 5x + 1 = 0$ by Chebyshev’s Method. (10)

(b). Solve the system by Relaxation method

$10x - 2y - 2z = 6,\ -x + 10y - 2z = 7,\ -x - y + 10z = 8$ (10)

9. (a).Evaluate $\int_0^6 \frac{dx}{1+x}$ by Trapezoidal rule, Simpson’s rule and Weddle’s rule. Also check up the results by actual integration. (10)

(b).Find the cubic spline approximation for the function given below.

x :	-1	0	1	2
y=f(x):	0	1	2	18

Assume M (0) = M (3) = 0. (10)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3010 Graph Theory and Algorithms

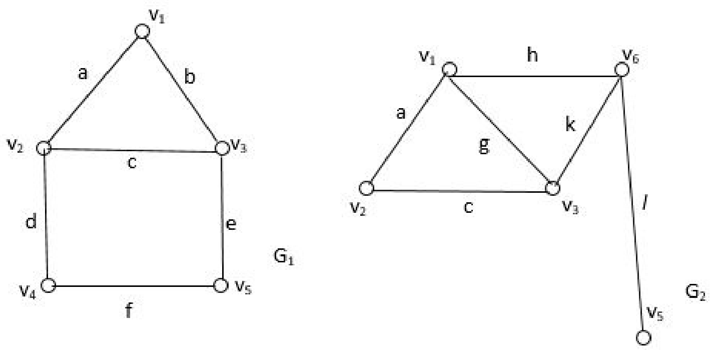
Set B

Time : 3 hrs
Total Marks: 100

1. (a) Prove that in a connected G with exactly 2k odd vertices, there exist k edge – disjoint subgraphs such that they together contain all edges of G that each is a unicursal graph. (10 marks)
(b) Prove that a graph G with n vertices, n – 1 edges and no circuits is connected. (10 marks)

OR

2. (a) For the given graphs find $G_1 \cup G_2$, $G_1 \cap G_2$, $G_1 \oplus G_2$ and delete vertex v_3 from $G_1 \cup G_2$ (10 marks)



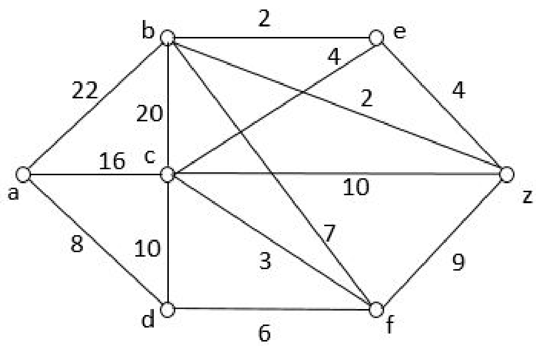
(b) Prove that every tree has either one or two centers. (10 marks)

3. (a) A simple graph with n vertices and k components can have at most (n – k) (n – k + 1) / 2 edges. (10 marks)
(b) State and prove Euler's Formula. (10 marks)

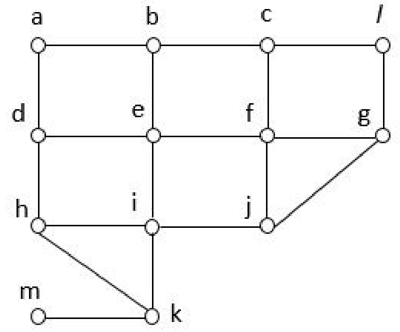
OR

4. (a) A connected graph G is an Euler graph if and only if it can be decomposed into circuits. (10 marks)
(b) Prove that a graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \ldots \ldots (\lambda - n + 1)$ (5 marks)
(c) Find the chromatic polynomial and the chromatic number of $K_{3,3}$ (5 marks)

5. (a) Use the Dijkstra's algorithm to find the shortest path between a to z. (15 marks)

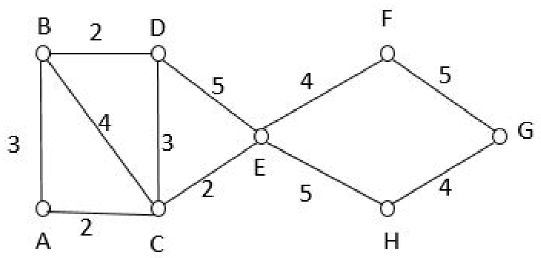


(b) Using Breadth First Search Algorithm find a spanning tree of the graph G (5 marks)



OR

6. Find the minimal spanning tree of the given graph using Prim's and Kruskal's Algorithm (20 marks)



7. (a) Solve the following LPP using Simplex method (10 marks)

$\min z = x_1 - 3x_2 + 2x_3$ Subject to the constraints

$3x_1 - x_2 + 2x_3 \leq 7$;

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$ and

$x_1, x_2, x_3 \geq 0$

(b) Solve the following LPP using graphical method (10 marks)

$\max z = 5x_1 + 8x_2$ Subject to the constraints

$15x_1 + 10x_2 \leq 180$;

$10x_1 + 20x_2 \leq 200$

$15x_1 + 20x_2 \leq 210$ and

$x_1, x_2 \geq 0$

OR

8. (a) Solve the following LPP using Simplex method (10 marks)

$\max z = 20x_1 + 6x_2 + 8x_3$ Subject to the constraints

$8x_1 + 2x_2 + 3x_3 \leq 250$;

$4x_1 + 3x_2 \leq 150$

$2x_1 + x_3 \leq 50$ and

$x_1, x_2, x_3 \geq 0$

(b) Solve the following LPP using graphical method (10 marks)

$\min z = -x_1 + 2x_2$ Subject to the constraints

$-x_1 + 3x_2 \leq 10$;

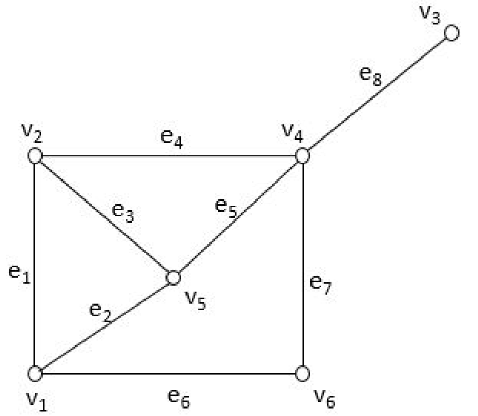
$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$ and

$x_1, x_2 \geq 0$

9. Find the (i) adjacency matrix X of the graph G given below and give five observations on it.

(ii) Find X and X^2 and give their properties. (20 marks)



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End Semester Examinations - Nov-Dec 2015 Exams

14MA3011 Biostatistics and Quality Control

Set B

Time : 3 hrs
Total Marks: 100

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1. a) Find mean, median and mode for the following data (10)

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	28	125	270	303	197	65	10

b) Calculate standard deviation and mean deviation from the following data (10)

X	10-15	15-20	20-25	30-35	40-45	50-55
f	8	12	15	10	3	2

OR

2. a) Calculate coefficient of quartile deviation and coefficient of variation from the following data:(10)

Marks	No. of Students
0 – 20	8
20 - 40	12
40 - 60	30
60 - 80	20
80 - 100	10

b) (i)The population (in millions) in the first eight censuses was as follows: (5)

3.9,5.3,7.2,9.6,12.9,17.1,23.2,31.4.Find the geometric mean.

(ii) Find the harmonic mean of the following data: 3,14,25,36,47,58,69,80. (5)

3. a) The weekly wages of 1000 workmen are normally distributed with mean Rs.70 and standard deviation Rs 5. Estimate the number of workers, whose weekly wages will be (I)less than Rs. 69. (II)more than Rs.72. (III)between Rs 69 and Rs72. (10)

b) A set of 8 symmetrical coins were tossed 256 times and their frequencies of throws observed were as follows: Fit a binomial distribution to this data and calculate the expected frequencies. (10)

No.of heads	0	1	2	3	4	5	6	7	8
Frequency	2	6	24	63	64	50	36	10	1

OR

4. a) In a large consignment of electric bulbs, 5% are defective. A random sample of 15 is taken for inspection. Use Binomial Distribution to find the probability that (i) all are good bulbs

(ii)exactly 3 are defective bulbs.(iii)atmost 3 are defective bulbs (iv)atleast 3 are defective bulbs (10)

b) Fit a Poisson distribution to the given data and calculate the expected frequencies. (10)

x	0	1	2	3	4
f	43	38	22	9	1

5. a) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal.

Test the hypothesis that proportions of men and women in favour of the proposal are same against that they are not, at 5% level. (10)

b) The means of two large samples of 1000 and 2000 members are 67.5 inches and 68 inches respectively. Can the samples be regarded as drawn from the same population of

standard deviation of 2.5 inches? Test at 5% level of significance. (10)

OR

6. a) Two samples are drawn from two normal population. From the following data, test whether the two samples have the same variance at 5 % level: (10)

Sample 1	60	65	71	74	76	82	85	87		
Sample 2	61	66	67	85	78	63	85	86	88	91

b) A manager of pizza hut was interested to determine whether sales of pizza is greater on one day of the week than another. His records from the past, shows the following result.Test whether sales of pizza is uniformly distributed over the week. (10)

Days of the week	MON	TUE	WED	Thurs	FRI
Number of pizza's sold	66	57	54	48	75

7.

a) Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover.The results are as follows(recovery time in days)

Treatment				
Doctor	1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Discuss the difference between (i) doctors and (ii) treatments (15)

b) Explain the aim of Design of Experiments. (5)

OR

8.

a) Five varieties of wheat A, B, C, D and E were studied. The plan, the varieties shown in each plot and yields obtained in kg are given in the following table.

B(90)	E(80)	C(134)	A(112)	D(92)
E(85)	D(84)	B(70)	C(141)	A(82)
C(110)	A(90)	D(87)	B(84)	E(69)
A(81)	C(125)	E(85)	D(76)	B(72)
D(82)	B(60)	A(94)	E(85)	C(88)

Perform the analysis of variance using Latin square.

9. a) Ten pieces of cloth out of different rolls of equal length contained the following number of defects:1, 3, 5, 0, 6, 0, 9, 4, 4, 3 . Draw a control chart for the number of defects and state whether

the process is in a state of statistical control. (10)

b) The following is a double sampling plan: (10)

$N=2000, n_1=50, c_1=2, n_2=60, c_2=5$ where N= the size of the lot, n_1= the size of the first sample, c_1= the maximum allowable number of defectives for acceptance on the basis of the first sample, n_2= the size of the second sample, c_2 = the maximum allowable number of defectives for acceptance on the basis of two samples. Interpret the above plan and point out its superiority over a single sampling plan.

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3014 Fundamentals of Statistics

Set B | **Time : 3 hrs**
Total Marks: 100

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1. a) Find mean, median and mode for the following data (10marks)

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	2	28	125	270	303	197	65	10

b) The scores of two batsmen A and B in 10 innings during a certain session are given below.Determine which of them is more consistent.

A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

OR

2. The following table gives the aptitude test scores and productivity indices of 10 workers selected random:

Aptitude Scores (X):	60	62	65	70	72	48	53	73	65	82
Productivity Index (Y):	68	60	62	80	85	40	52	62	60	81

Calculate the two regression equations and estimate (i) the productivity index of a worker whose test score is 92. (ii)the test score of a worker whose productivity index is 75
.

3. a) The first bag contains 3 white balls, 2 red balls and 4 black balls. Second bag contains 2 white, 3 red and 5 black balls and Third bag contains 3 white, 4 red and 2 black balls. One bag is chosen at random and from it 3 balls are drawn. Out of three balls two balls are white and one is red. What are the probabilities that they were taken from first bag, second bag and

third bag. (10)

b) Two fair dice are thrown independently. Three events A,B,C are defined as follows,

A. odd face with the first die

B. odd face with the second die

C. sum of the numbers in the 2 dice is odd. Are the events A,B,C mutually independent (10)

OR

4. a) A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random. Find the probability that (i) both have major defects (ii) both are good. (iii) atleast 1 is good. (iv) neither has major defects (v) atmost 1 is good. (10)

b) Players X and Y roll a pair of dice alternately. The player who rolls 11 first wins. If X starts the game, find his chance of winning. (10)

5. a) The distribution of typing mistakes committed by a typist is given below .Fit a Poisson Distribution to the given data and find out the expected frequencies: (10)

No. of mistakes per page	0	1	2	3	4	5
No. of pages	142	156	69	27	5	1

b) 1000 light bulbs with a mean life of 120 days are installed in a new factory; their length of life is normally distributed with standard deviation 20 days. How many bulbs will expire in

(i) less than 90 days (ii) lmore than 90 days(iii) between 75 and 90days? (10)

OR

6. a) Suppose that a manufactured product has 2 defects per unit of product inspected. Using Poisson distribution, calculate the

probabilities of finding a product(i) without any defect,

(ii) 3 defects or 4 defects (iii) exactly 2 defects. (10)

b) A set of 8 symmetrical coins were tossed 256 times and their frequencies of throws observed were as follows

No.of heads	0	1	2	3	4	5	6	7	8
Frequency	2	6	24	63	64	50	36	10	1

Fit a binomial distribution to this data and calculate the expected frequencies. (10)

7. a) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test

the hypothesis that proportions of men and women in favour of the proposal are same against that they are not, at 5% level. (10)

b) The means of two large samples of 1000 and 2000 members are 67.5 inches and 68 inchesrespectively. Can the samples be regarded as drawn from the same population of standard

deviation of 2.5 inches? Test at 5% level of significance. (10)

OR

8. a)Two samples are drawn from two normal population. From the following data test whether the two samples have the same variance at 5 % level: (10)

Sample 1	23	24	25	26	27	
Sample 2	22	27	28	30	31	36

8.b The following data are collected on two characters

	Smokers	Non smokers
Literates	83	57
Illiterates	45	68

Bsaed on this,can you say that there is no relation between smoking and literacy? (10)

.

9. a) The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machines: (15)

		Machine Type			
		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the five men differ with respect to mean productivity.

(ii) Test whether the mean productivity is the same for the four different machine types.

b) Compare RBD and LSD (5)

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End Semester Examinations - Nov-Dec 2015 Exams

14MA3015 Operations Research Techniques

Set B

Time : 3 hrs
Total Marks: 100

1. Solve by two phase simplex method

Maximize $Z=5x_1 + 8x_2$

Subject to

$3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

and $x_1, x_2 \geq 0$.

OR

2. (a) A company manufactures 2 types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with other data are given below.

	•		Stock Available
	A.	A.	
◦	1.	1.	1.
◦	1.	1.	1.
◦	1.	1.	1.
◦	1.	1.	

Using graphical method find how many circuits of each type should the company produce from the stock to earn maximum profit. (12)

(b) Solve the following LPP by simplex method. (8)

Minimize $Z = 8x_1 - 2x_2$

Subject to

$-4x_1 + 2x_2 \leq 1$

$5x_1 - 4x_2 \leq 3$

and $x_1, x_2 \geq 0$.

3. (a) Determine the optimal sequence of jobs that total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed. (14)

	Machines			
Jobs	M_1	M_2	M_3	M_4
A	13	8	7	14
B	12	6	8	19
C	9	7	8	15

D	8	5	6	15

(b) There are five jobs, each of which is to be processed through two machines M_1, M_2 in the order M_1M_2. Processing hours are as follows:

Job	1	2	3	4	5
M_1	3	8	5	7	4
M_2	4	10	6	5	8

Determine the optimum sequence for the five jobs. (6)

OR

4. (a) Find the initial basic feasible solution in the following transportation problem by Vogel's approximation method. Also obtain the optimum solution. (14)

	1	2	3	Supply
I	7	3	2	2
II	2	1	3	3
III	3	4	6	5
Demand	4	1	5	10

(b) Determine basic feasible solution to the following transportation problem using North West Corner Rule: (6)

	A	B	C	D	E	Supply
I	2	11	10	3	7	4
II	1	4	7	2	1	8
III	3	9	4	8	12	9
Demand	3	3	4	5	6	21

5. .(a) A company faced problem of assigning 5 jobs to 5 persons. The assignment costs are given as follows:

	Job				
Person	J_1	J_2	J_3	J_4	J_5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule. (10)

(b) A machine shop purchased a drilling machine and two lathes of different capacities. The positioning of the machines among 4 possible locations on the shop floor is important from the standard of materials handling. Given the cost estimate per unit time of materials below, determine the optimal location of the machines. (10)

	Location			
	1	2	3	4
Lathe 1	12	9	12	9
Drill	15	Not suitable	13	20
Lathe2	4	8	10	6

OR

6. . (a) In a railway Marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time

follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following:

(i) the mean Queue size,

(ii) the probability that Queue size exceeds 10. (10)

(b) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.

(i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?

(ii) If the subscribers will wait and are serviced in turn, what is the expected waiting time? (10)

7. Construct the network for the project whose activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and the project duration.

Activity	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration (in weeks)	6	5	10	3	4	6	2	9

OR

8. The following time - cost table (time in days, cost in rupees) applies to a project. Determine the network to complete the project in minimum time at minimum cost.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	2	800	1	1400
1-3	5	1000	2	2000
1-4	5	1000	3	1800
2-4	1	500	1	500
2-5	5	1500	3	2100
3-4	4	2000	3	3000
3-5	6	1200	4	1600
4-5	3	900	2	1600

9. The occurrence of rain in a city on a day is dependent upon whether it is rained or not on the previous day. If it rained on the previous day, the distribution is,

Event	No rain	1cm rain	2cm rain	3cm rain	4cm rain	5cm rain
Probability	0.5	0.25	0.15	0.05	0.03	0.02

If it did not rain on the previous day, the rain distribution is,

Event	No rain	1cm rain	2cm rain	3cm rain
Probability	0.75	0.15	0.06	0.04

Simulate the city's weather for 10 days and determine the total days without rain as well as the total rainfall during the period. Use the following random numbers for simulation:

67, 63, 39, 55, 29, 78, 70, 06, 78, 76. Assume that for the first day of simulation it had not rained the day before.

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End Semester Examinations - Nov-Dec 2015 Exams

15MA3001 Algebra

Set B

Time : 3 hrs
Total Marks: 100

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1. a. State and prove fundamental theorem of arithmetic. (12 Marks)

   b. Let a and b be integers not both zero. Then a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$. (8 Marks)

OR

2. a. If $a \mid c$ and $b \mid c$ with $gcd(a,b) = 1$, then $ab \mid c$. (8 Marks)
   b. If $a \mid bc$ with $gcd(a,b) = 1$, then $a \mid c$. (8 Marks)
   c. Find $gcd(1769, 2378)$ using Euclidean algorithm and then write gcd as a linear combination of 1769 and 2378. (4 Marks)

3. a. If p is a prime, then $(p-1)! \equiv -1 (mod\ p)$. (10 Marks)
   b. Let p be a prime and suppose that p does not divides a. Then $a^{p-1} \equiv 1 (\text{mod } p)$. (10 Marks)

OR

4. a. If the integer a has order k *modulo* n, then $a^i \equiv a^j (\text{mod } n)$ if and only if $i \equiv j (\text{mod } k)$. (8 Marks)

   b If p is a prime and $d \mid p-1$, then the congruence $x^d - 1 \equiv 0 (\text{mod } p)$ has exactly d solutions. (12 Marks)

5. Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate set S.

OR

6. a. Let G be a group of order 351. Then prove that G has a normal *Sylow p-subgroup* for some prime p dividing 351. (10 Marks)

   b. State and prove *Second Part of Sylow's Theorem.* (10 Marks)

7. Prove that every finite abelian group is the direct product of cyclic groups.

OR

8. a. If R is a ring, then prove that for all a, b in R, (10 Marks)

   a. $0a = a0 = 0$.

   b. $(-a)b = a(-b) = -(ab)$

   c. $(-a)(-b) = ab$

   If, in addition, R has a unit element, then

   d. $(-1)a = -a$

   e. $(-1)(-1) = 1$.

   b. Let R be a commutative ring with unit element whose only ideals are $\{0\}$ and R. Then prove that R is a field. (10 Marks)

9. a. Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists a in A such that A consists of all ax as x ranges over R. (10 Marks)

   b. Let R be a Euclidean ring. Then prove that any two elements a and b in R have a greatest common divisor d. (10 Marks)

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End Semester Examinations - Nov-Dec 2015 Exams

15MA3002 Ordinary Differential Equations

Set A | **Time : 3 hrs**
Total Marks: 100

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1. (a). Let A(t) be a n×n matrix that is continuous in t on a closed and bounded interval I. Suppose a Matrix Φ satisfies the matrix differential equation $X' = A(t)X, t\in I$. Then show that det Φ satisfies the first order equation $(\det \Phi)' = (\text{tr } A)(\det \Phi)$. (10marks)

(b). Show that a solution Φ of $X' = A(t)X, t\in I$, is a fundamental matrix of $x' = A(t)x$ on I if and only if $\det \Phi(t) \neq 0, t\in I$. (5marks)

(c). Let $\Phi(t)$ denote a fundamental matrix of of the system $x' = Ax$ where A is a constant matrix. Then show that $\Phi(t+s) = \Phi(t)\Phi(s)$ for all values of t,s in I. (5marks)

OR

2. (a). Let $\Phi(t)$ denote a fundamental matrix of the system $x' = A(t)x$ on I. Then show that the solution F(t) of the IVP $x' = A(t)x + b(t), x(t_0) = x_0$ is given by

$$F(t) = \Phi(t)\Phi^{-1}(t_0)x_0 + \int_{t_0}^{t} \Phi(t)\Phi^{-1}(s)b(s)ds. \quad (10\text{marks})$$

(b). Determine exp(tA) for the system $x' = Ax$ where

$$A= \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}. \qquad (10\text{marks})$$

3. (a). Define Lipschitz condition. (2marks)

(b). Let f(t, x) be a continuous function defined over a closed rectangle R. Show that if the partial derivative of f wrt x exists and is continuous on R then f satisfies the Lipschitz condition in R. (8marks)

(c). State and prove Gronwall Inequality. (8marks)

(d). Let f(t) be non-negative continuous function for $t\geq t_0$.

If $f(t)\leq k \int_{t_0}^{t} f(s)ds$ then show that $f(t) = 0$ for $t\geq t_0$. (4marks)

OR

4. State and prove Picard's theorem for the existence of a unique solution for a class of non-linear Initial Value Problems.

5. (a). Define equicontinuous family of functions. (2marks)

(b) . State Ascoli's Lemma. (2marks)

(c). Establish an existence of a solution for the Initial Value Problem $x' = f(t, x)$, $x(t_0) = x_0$ where $f \in C[D,R]$, D is an open connected set in R^2 and $(t_0,x_0) \in D$. (16marks)

OR

6. State and prove Alekseev's Formula for a solution of the perturbed system $y = f(t, y) + F(t, y)$, $y(t_0) = x_0$, $t \geq t_0$.

7. Explain Sturm –Liouville Problem. Let x_m and x_n be the eigen functions of the Sturm –Liouville Problem corresponding to two eigen values λ_m, λ_n . Then establish a relation connecting the Wronskian $W(x_m, x_n)$.

OR

8. Explain briefly the application of boundary Value Problem to a physical problem in engineering.

9. State and prove Sturm's Comparison theorem.

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End Semester Examinations - Nov-Dec 2015 Exams

15MA3003 Classical Mechanics

Set B **Time : 3 hrs**
Total Marks: 100

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1. (a).Find the Jacobian transformation for a particle which is constrained to move on a fixed circular path **a** . (8)

   (b). Derive De Alembert's principle. (12)

OR

2. (a). Define virtual work and virtual displacement. (4)

   (b). Derive Lagrangian Equation for a system of N particles. (16)

3. (a).Define Ruthian function (4)

   (b).Two Particles of mass m_1 and m_2 are connected by a light string of length l which posses over a smooth pulley. Obtain equation of motion. (16)

OR

4. (a).Prove that the generalized momentum corresponding to each ignorable coordinate is constant. (4)

   (b).Derive Hamilton's canonical equation. (16)

5. (a).Define Normal coordinates. (4)
   (b).Discuss Kepler's problem using Hamilton's function. (16)

OR

6. (a).Define Normal coordinates (4)
   (b).Obtain Hamilton's equation for the mass string problem. (16)

7. (a).Define natural system (4)
   (b).Derive Principle of least action. (16)

OR

8. (a).Define stationary value of a function (4)
   (b).Show that ω is a constant if $\vec{G}=0$ and $I_{xx}=I_{yy}$ under Euler dynamical equation of motion. (16)

9. (a). A slender bar of length l and mass m slides on the smooth floor and wall and has counter clockwise angular velocity $\vec{\omega}$ .What is the acceleration of bar? (10)

   (b). A body turns about a fixed point. Show that the angle between its angular velocity and angular momentum about a fixed point is acute. (10)

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End Semester Examinations - Nov-Dec 2015 Exams

15MA3004 Real Analysis

Set A | **Time : 3 hrs** | **Total Marks: 100**

1.

a) State and prove the unique factorization theorem. (10)

b) If $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$, then show that number 'e' is irrational. (10)

OR

2.

a) State and prove Archimedean property of real number system. (10)

b) If $a \geq 0$, then prove that $|x| \leq a$ if and only if $-a \leq x \leq a$. (4)

c) For arbitrary real x and y, show that $|x+y| \leq |x| + |y|$. (6)

3.

a) Prove that the set of all real numbers is uncountable. (10)

b) If F is a countable collection of disjoint sets, say $F = \{A_1, A_2, \ldots\ldots\}$, such that each set A_n is countable, and then prove that the union $\bigcup_{k=1}^{\infty} A_k$ is also countable. (10)

OR

4.

a) Let Z^+ denote the set of all positive integers. Then show that the Cartesian product $Z^+ \times Z^+$ is countable. (10)

b) Let F be a collection of sets then for any set B, show that

$$i)\ B - \bigcup_{A \in F} A = \bigcap_{A \in F} (B - A)$$
$$ii)\ B - \bigcap_{A \in F} A = \bigcup_{A \in F} (B - A)$$
(10)

5.

a) Define open set and give an example. Also prove the union of any collection of open sets is open. (8)

b) State and prove Cantor intersection theorem. (12)

OR

6.

a) If '$\bar{x}$' is an accumulation point of S, then prove that every n-ball $B(\bar{x})$ contains infinitely many points of S. (8)

b) State and prove Heine-Borel covering theorem. (12)

7. a) State and prove fixed point theorem for contractions. (12)

b) Prove that $f(x)=\begin{cases}\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ is continuous on $(0,\infty)$ but not uniformly continuous on $(0,\infty)$. (8)

OR

8. a) State and prove Rolle's Theorem. (10)

b) Let *f* is the function defined on *(a, b)*, then prove that there is the function f^* which is continuous at *c* and which satisfies the equation, $f(x)-f(c)=(x-c)f^*(x)$, for all *x* in *(a, b)*, with $f'(c)=f^*(c)$. Conversely, if there is a function f^*, continuous at *c*, then prove that *f* is differentiable at *c* and $f'(c)=f^*(c)$. (10)

9. Let each term of $\{f_n\}$ is a real-valued function having a finite derivative at each point on *(a, b)* and for at least one point x_0 in *(a, b)* the sequence $\{f_n(x_0)\}$ converges. And let there exists a function *g* such that $f_n' \to g$ uniformly on *(a,b)*. Prove that

i) There exists a function *f* such that $f_n \to f$ uniformly on *(a,b)*.

ii) For each *x* in *(a,b)* the derivative $f'(x)$ exists and equal to $g(x)$.

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End Semester Examinations - Nov-Dec 2015 Exams

15MA3017 Mathematics for Competitive Examinations

Set A

Time : 3 hrs
Total Marks: 100

1. 1(a). Find the L.C.M of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{9}{13}$.

(b). Arrange the fractions $\frac{3}{5}, \frac{4}{7}, \frac{8}{9}$ *and* $\frac{9}{11}$ in their descending order.

(c). Find the square root of 1471369.

(d). $\left(\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right) \div \sqrt{\frac{16}{81}} = ?$

OR

2. 2(a). $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3} = ?$.

(b).If $a^2+b^2 = 117$ and $ab = 54$, then find the value of $\frac{a+b}{a-b}$.

(c). Find the value of $\left(\frac{0.1\times0.1\times0.1+0.02\times0.02\times0.02}{0.02\times0.02\times0.02+0.04\times0.04\times0.04}\right)$

(d). Find the least value of * for which 7*5462 is divisible by 9.

3. 3 (a). Distance between two stations A and B is 778km. A train covers the journey from A to B at 84km per hour and returns back to A with the uniform speed of 56km per hour. Find the average speed of the train during the whole journey.

(b) The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

(c). Rohit was 4 times as old as his son 8 years ago. After 8 years Rohit will be twice as old as his son. What are their present ages?.

(d). In an examination, 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.

OR

4. 4(a). By selling 33metres of cloth, one gains the selling price of 11metres. Find the gain percent.

(b). If x:y = 3:4, find (4x+5y) : (5x-2y).

(c). A and B can do a piece of work in 18 days; B and C can do it in 24 days; A and C can do it 34 days. In how many days will A,B and C finish it, working together and separately?.

(d). Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

5. 5(a). A man travelled from the village to the post office at the rate of 25kmph and walked back at the rate of 4kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post office from the village.

(b). Two trains 100metres and 120metres long are running in the same direction with the speeds of 72kmph and 54kmph. In how much time will the first train cross the second?.

(c). There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12kmph, while the other sails on a boat at a speed of 10kmph. If the river flows at the speed of 4kmph, which of the two friends will return to place A first?.

(d). In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price?

OR

6. 6 (a). What was the day of 24 May 1988 ?.

(b). Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.

(c). Find the income derived from 88 shares of Rs.25 each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}$% per annum. Also, find the rate of interest on the investment.

(d). How many words can be formed from the letters of the word 'DIRECTOR' so that the vowels are always together?.

7. 7 (a). In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi?.

(b). A clock is set right at 5a.m. The clock loses 16minutes in 24 hours. What will be the true time when the clock indicates 10p.m. on the 4th day?.

(c). A sells Rs.5000, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He there by increases his income by Rs.70. How much of the proceeds were invested in each stock?.

(d). How many 3-digit numbers can be formed from the digits 2,3,5,6,7 and 9, which are divisible by 5 and none of the digits is repeated?.

OR

8. 8 (a). A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?.

(b). A train 100m long is running at the speed of 30kmph. Find the time taken by it to pass a man standing near the railway line.

(c). A man takes 3hours 45minutes to row a boat 15km downstream of a water and 2hours 30minutes to cover a distance of 5km upstream. Find the speed of the river current in kmph.

(d) . In what ratio must rice at Rs.9.30 per kg be mixed with rice at Rs.10.80 per kg so that the mixture be worth Rs.10 per kg?.

9. 9(a). 9^3 is subtracted from the square of a number , the answer so obtained is 567. What is the number ?

(b). Pinku, Rinku and Tinku divide an amount of Rs.4200 amongst themselves in the ratio of 7:8:6 respectively. If an amount of Rs.200 is added to each of their shares, what will be the new respective ratio of their shares of amount?

(c). The product of two successive numbers is 8556. What is the smaller number?

(d). What is the least number to be added to 4321 to make it a perfect square?

Wishing you All the Best

Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

End Semester Examinations - Nov-Dec 2015 Exams

15MA3018 Probability and Distributions

Set B — **Time : 3 hrs**
Total Marks: 100

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1. a) State and prove Baye's theorem. (8)

   b)There are 4 white balls and 5 black balls in a bag.3 balls are successively drawn at random without replacement,what is the probability that the first draw to give 2 white balls and the second draw to give one white ball? (6)

   c) The PMF of a random variable X is given by $P(x)=k(2/3)^{x}$,x=1,2,3.(i)Find the value of k (ii)Find the mean and variance of X.(6)

OR

2. a). State and prove Tchebycheff's inequality.(8)

   b).Let X be the life time of a mechanical part.Assume that X has the CDF,F(x)=0 if x<0 and $F(x)=1-e^{-x}$,if $x\geq 0$.(i) Find the PDF of X.(ii)Also find P(1<X<3).(6)

   c) If X and Y are independent random variables,prove that E(XY)=E(X) E(Y).(6)

3. (a) The joint probability density function of a two dimensional variable (X, Y) is $f(x,y)=4xy, 0<x<1, 0<y<1$. Find E[X], E[X^2], E[XY] and $E[3Y-2X^2+6XY]$. Are X and Y are independent variable ? (10)

   (b) Let X_1 and X_2 have the joint the joint probability density function $f(x_1,x_2)=10x_1x_2^2$, $0<x_1<x_2<1$. Let $Y_1=\frac{X_1}{X_2}$ and $Y_2=X_2$ Find the joint probability density function of (Y_1, Y_2)and marginal probability density function of Y_2. (10)

OR

4. (a) Let X_1 and X_2 have the joint the joint probability mass function given by the following table.

(X_1, X_2)	(0, 0)	(0, 1)	(0, 2)	(1, 1)	(1, 2)	(2, 2)
$p(x_1, x_2)$	1/12	2/12	1/12	3/12	4/12	1/12

   Find the correlation coefficient. (10)

   (b) The joint probability density function of a two dimensional variable (X, Y) is $f(x,y)=\frac{x+3y^2x}{4}, 0<x<2, 0<y<1$. Find the (i) the marginal density function of X and Y (ii) $f(x/y)$ (iii) $P(\frac{1}{4}<X<\frac{1}{2}/Y\leq\frac{1}{3})$. (10)

5. a) Find the MGF of chi-square distribution. (14)

   b).If X has the pdf $f(x)=xe^{-x/2}$,0<x< ∞,find the MGF of X. Also find μ and σ^2 (6)

OR

6. a) Find the MGF of normal distribution. (14)

   b) Let Y be the number of success in n-independent trials of a random experiment having the probability of success p=2/3.(i) if n=3,compute P(2≤Y) (ii)if n=5,compute P(3≥ Y). (6)

7. Derive the probability distribution function of F distribution. (20)

OR

8. Derive the probability density function of Beta distribution. (20)

9. a) State and prove Central limit theorem. (14)

b) A random variable X_n (n=1,2,3,…)are independent and each of them has the poisson distribution $P(X_n=r)=2^r e^{-2}/r!$,r=0,1,2,3,… and let $Y_{100}=X_1+X_2+\ldots\ldots+X_n$. Find $P(190 < Y_{100} < 210)$. (6)

Wishing you All the Best
