Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

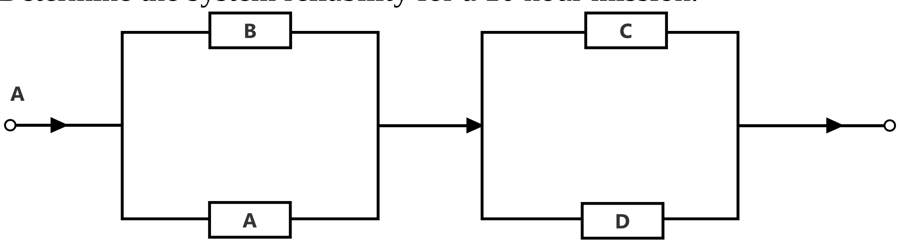
End Semester Examination – April/May – 2017

Code : 15MA3019 **Duration : 3hrs**

Sub. Name : Stochastic Processes **Max. marks : 100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	Given a random variable with the characteristic function $\phi(\omega)=E\left\{e^{i\omega Y}\right\}$ and a random process defined by $X(t)=\cos(\lambda t+Y)$, show that $\{X(t)\}$ is wide sense stationary.	CO 1	10
	b.	If $U(t)=X\cos t+Y\sin t$ and $V(t)=Y\cos t+X\sin t$, where X and Y are independent random variables such that $E(X)=0=E(Y)$ and $E(X^2)=E(Y^2)=1$, show that $\{U(t)\}$ and $\{V(t)\}$ are individually stationary in wide-sense, but they are not jointly wide sense stationary.	CO 1	10
(OR)				
2.	a.	A gambler has ₹ 2. He bets ₹ 1 at a time and wins ₹ 1 with probability $1/2$. He stops playing if he loses ₹ 2 or wins ₹ 4. a. What is the transition probability matrix of the related Markov processes? b. What is the probability that he has lost his money at the end of 5 plays? c. What is the probability that the game lasts more than 7 plays.	CO 1	10
	b.	A man either drives a car or goes by train to office. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Suppose on the first day of the week, the man tosses a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes train on the third day and the probability that he drives to work on the long run	CO 1	10
3.	a.	Prove that the sum of two Poisson processes is a Poisson process and the difference of two Poisson processes is not a Poisson process?	CO 2	10
	b.	The demand of cycle on each day in a cycle hiring shop is Poisson distributed with mean 2. The shop has 3 cycles. Find the proportion of the days in which a. no cycle is used b. some demand of cycle is refused c. no request is refused	CO 1	10
(OR)				
4.	a.	Prove that the interarrival time of a Poisson process with parameter λ has an exponential distribution with mean $\dfrac{1}{\lambda}$.	CO 1	5
	b.	If the number of occurrences of an event E in an interval of length t is a Poisson process $\{X(t)\}$ with parameter λ and if each occurrences of E has a constant probability p of being recorded and the recordings are independent of each other, then show that the number $N(t)$ of the recorded occurrences in t is also a Poisson process with parameter λp.	CO 1	5

	c	If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is a. more than one minute b. between 1 min and 2 min c. 4 min or less d. more than 4 mins	CO 1	10
5.	a.	Define Birth – Death process and derive the probability distribution of the B – D processes.	CO 2	10
	b.	Define Erlang process and derive the mean and variance of the Erlang process.	CO 2	10
(OR)				
6.	a.	If the company employs n sales persons, its gross sales in thousands of rupees may be regarded an Erlang processes having $\lambda = \frac{1}{2}$ and $k = 80000\sqrt{n}$. If the sales cost is ₹ 8000.00 per sales person. How many sales persons should the company employ to maximize the expected profit.	CO 2	10
	b.	Consumer demand for milk in certain locality, per month, is known to be a general gamma random variable. If the average demand is a liters and the mostly likely demand is b liters $(b < a)$, what is the variance of the demand.	CO 2	10
7.	a.	A duplicate machine maintained for office use is operated by an office assistant who earns ₹ 5.00 per hour. The time to complete each job varies according to an exponential distribution with mean 6 min. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8-hour day is used as a base, determine a. the percentage idle time of the machine b. the average time a job is in the system and c. the average earning per day of the assistant	CO 3	10
	b.	Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels. Assume both queues to be of Poisson type.	CO 3	10
(OR)				
8.	a.	The reliability of a motor is given by $R(t) = \left(1 - \frac{t}{t_0}\right)^2, 0 \le t \le t_0$. a. Show that the motors are experiencing were out. b. Find the mean time to failure as a function of the maximum life. c. If the maximum life is 1500 operating hours, what is the design life for a reliability of 0.95?	CO 3	10
	b.	Consider the figure given. Assume that the components are functioning independent and follow Weibull failure distribution with parameter $\alpha = \beta = 1$ for A and B, $\alpha = 1$, $\beta = 0.5$ for C and $\alpha = 1$, $\beta = 2$ for D. Determine the system reliability for a 10 hour mission. 	CO 3	8

	c	A system has 4 identical components connected in parallel and shows a system reliability of 0.90. how many more components should be added in parallel to get a system reliability of 0.99?		2
		Compulsory:		
9.	a.	If the processes $\{X(t)\}$ is defined as $X(t) = Y(t)Z(t)$, where $\{Y(t)\}$ and $\{Z(t)\}$ are independent WSS process, with autocorrelation function $R_{xx}(\tau) = R_{yy}(\tau)R_{zz}(\tau)$, prove that, $S_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{yy}(\alpha)S_{zz}(\omega-\alpha)\,d\alpha$	CO 2	10
	b.	The power spectral density function of a zero mean WSS process $\{X(t)\}$ is given by $S(\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_0 \\ 0, & elsewhere \end{cases}$. Find $R(\tau)$ and show also that $X(t)$ and $X\left(t + \frac{\tau}{\omega_0}\right)$ are uncorrelated.	CO 2	10

ALL THE BEST