Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – April/May – 2017

Code : 15MA3010 **Duration : 3hrs**

Sub. Name : Advanced Calculus **Max. marks : 100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	If *f* monotonic on *[a, b]*, then prove that the set of discontinuities of *f* is countable .	CO 1	4
	b.	Let *f* be defined a function on *[a, b]*. Then prove that *f* is of bounded variation on *[a, b]* if and only if *f* can be expressed as the difference of two increasing functions.	CO 1	4
	c.	Assume that $c\ \varepsilon\ (a, b)$. If two of the integrals in below equation exist, then prove that the third also exists and $\int_a^c f + \int_c^b f = \int_a^b f$	CO 1	12
		(OR)		
2.	a.	Assume $f \varepsilon R(\alpha)$ on *[a,b]* and assume that α has a continuous derivative $\alpha'(x)$ on *[a, b]*. Then prove that the Riemann integral $\int_a^c f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.	CO 1	15
	b.	Assume that $\alpha\uparrow$ on *[a, b]*. Then prove that $\underline{I}(f,\alpha) \leq \bar{I}(f, \alpha)$.	CO 1	5
3.	a.	Assume that α is of bounded variation on *[a, b]*. Let $V(x)$ denote the total variation of α on *[a, x]* if $a < x \leq b$, and $V(a) = 0$. Let *f* be defined and bounded on *[a, b]*. Then prove that if $f\ \varepsilon\ R(\alpha)$ on *[a, b]*, then $f\ \varepsilon\ R(V)$ on *[a, b]*.	CO 1	15
	b.	Assume that $\alpha\uparrow$ on *[a, b]*. If $f\varepsilon R(\alpha)$ on *[a,b]* and $g\varepsilon R(\alpha)$ on *[a,b]* and if $f(x) \leq g(x)$ for all *x* in *[a, b]*, then prove that $\int_a^b f(x)d\alpha(x) \leq \int_a^b g(x)d\alpha(x)$.	CO 1	5
		(OR)		
4.	a.	State and prove the *First Mean – Value Theorem for Riemann – Stieltjes Integrals.*	CO 1	12
	b.	Let α be of bounded variation on *[a, b]*. Assume that each term of the sequence $\{f_n\}$ is a real valued function such that $f_n\ \varepsilon R(\alpha)$ on *[a, b]* for each $n = 1, 2,\ldots,$. Assume that $f_n \rightarrow f$ uniformly on *[a, b]* and define $g_n(x) = \int_a^x f_n(t)d\alpha(t)$ if $x\ \varepsilon\ [a, b]$, $n = 1,2,\ldots$. Then prove that i. $f\varepsilon R(\alpha)$ *on [a, b]*. ii. $g_n \rightarrow g$ uniformly on *[a, b]*, where $g(x) = \int_a^x f\ (t)d\alpha(t)$.	CO2	
5.	a.	If *f* and *g* are in *L(I)*, then prove that f^+, f^-, $\lvert f\rvert$, *max(f, g)* and *min(f, g)* are in *L(I)*. Also $\lvert\int_I f\rvert \leq \int_I \lvert f\rvert$.	CO1	12
	b.	Assume that $f, g\ \varepsilon\ U(I)$. Then prove that i. $(f+g)\ \varepsilon\ U(I)$ and $\int_I (f+g) = \int_I f + \int_I g$. ii. $cf\ \varepsilon\ U(I)$ for every constant $c \geq 0$, and $\int_I cf = c\int_I f$. iii. $\int_I f \leq \int_I g$ if $f(x) \leq g(x)$ almost everywhere on *I*. iv. $\int_I f = \int_I g$ if $f(x) = g(x)$ almost everywhere on *I*.	CO1	8
		(OR)		

6.	a.	State and prove the *Levi theorem for series of Lebesque-integrable functions.*	CO 3	
7.	a.	Let $\{f_n\}$ be a Cauchy sequence of complex valued functions in $L^2(I)$. Then prove that there exists a function in $L^2(I)$ such that $\lim_{n\to\infty} \|\|f_n - f\|\| = 0$.	CO3	20
(OR)				
8.	a.	Assume that f is differentialble at c with total derivative T_c. Then prove that the directional derivative f '(c;u) exists for every u in Rn and we have $T_c(u) = f\,'(c,u)$.	CO4	5
	b.	Let $f: S \rightarrow R^m$ be differentialble at an interior point c of S, where S is a subset of R^n. If $v = v_1u_1+\ldots+v_nu_n$, where $u_1,u_2,\ldots,u_n$ are the unit coordinate vectors in R^n, then prove that $f\,'(c) = \sum_{k=1}^{n} v_k D_k\ f(c)$	CO4	15
		<u>Compulsory:</u>		
9.	a.	State and Prove *the Implicit Function Theorem.*	CO 4	20

Course Outcomes: Students will be able to strengthen their knowledge in

CO 1: Differentiation & Integration,

CO 2: Uniform Convergence,

CO 3: Convergence Theorems,

CO 4: Inverse & Implicit Function Theorems.

ALL THE BEST