Reg.No. \_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – April/May – 2017**

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| **Code :** | **15MA3006** | **Duration :** | **3hrs** |
| **Sub. Name :** | **LINEAR ALGEBRA** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| Q. No. | Sub Div. | Questions | Course  Outcome | Marks |
| 1. | a. | If V be a vector space which is spanned by a finite set of vectors. Then prove that any linearly independent set of vectors in V is finite and contains no more than m elements. | CO1 | 10 |
| b. | Show that the vector = (1, 0, -1), = (1, 1,1) and= (1, 0, 0) form the basis for R4. Find the co-ordinates of each of the standard basis vectors in the order basis {,,}. | CO1 | 10 |
| (OR) | | | | |
| 2. | a. | If V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite-dimensional. Then prove that  Rank (T) + nullity (T) = dim V | CO1 | 10 |
| b. | If  be the linear transformation defined by T(x, y, z) = (x + 2y - z, y + z, x + y - 2z). Compute basis and the dimension of i)the image of T ii) the kernel of T. | CO1 | 10 |
| 3. | a. | Is there is the linear transformation from  such that T(1, 2) = (3, 2, 1) and T(3, 4) = (6, 5, 4). | CO1 | 10 |
|  | b. | If V be an m-dimensional vector space over the field F and W be an n-dimensional vector space over F. Then prove that the space L(V, W) is the finite dimensional and has dimension ‘mn’. | CO1 | 10 |
| (OR) | | | | |
| 4. | a. | If T be a linear operator on  defined by    i) What is the matrix form of T in the standard ordered basis for.  ii) What is the matrix form of T in the ordered basis {, , }, where = (1, 1, 1) , = (1, 1, 0) , = (1, 0, 0). | CO2 | 10 |
|  | b. | State and prove Cayley-Hamilton theorem. | CO2 | 10 |
| 5. | a. | If W be invariant subspace for T, then prove that the characteristic polynomial for the operator Tw divides the characteristic polynomial for T. Also prove that every minimal polynomial for Tw divides a minimal polynomial for T. | CO2 | 10 |
|  | b. | Suppose that the characteristic and minimal polynomial of the linear operator T are  and  respectively. Calculate all the possible Jordan canonical form of T. | CO2 | 10 |
| (OR) | | | | |
| 6. | a. | If V be a finite dimensional vector space over F and T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial of T is as the form where  are distinct elements of F. | CO2 | 10 |
|  | b. | If V be a vector space with dimension 6 over R and if T be a linear operator whose minimal polynomial is , Obtain Rational canonical form of T. | CO2 | 10 |
| 7. | a. | If A and B be two square matrices of order n over a field F and. Prove that  i)  ii)  iii) | CO3 | 8 |
|  | b. | If be linear. Let W be a T-invariant subspace of V and  the induced operator on *V/W*. Prove that -annihilator of divides the minimal polynomial of T. | CO3 | 12 |
| (OR) | | | | |
| 8. | a. | Prove that the quadratic form  is indefinite. | CO3 | 10 |
|  | b. | Obtain the matrix of the quadratic form  and verify that it can be written as matrix product. | CO3 | 10 |
|  | | **Compulsory**: |  |  |
| 9. | a. | State and prove Sylvester’s Law in nullity. | CO3 | 10 |
|  | b. | Reduce the following quadratic form to real canonical form and compute its rank and signature: | CO3 | 10 |

ALL THE BEST