Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

			Semester	:	**2016-17 ODD**
Code	:	**15MA3004**	**Duration**	:	**3hrs**
Sub. Name :		**Real Analysis**	**Max. marks :**		**100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	Prove that every integer $n>1$ is either a prime or a product of primes.	CO1	6
	b.	State and prove Euclid's Lemma.	CO1	6
	c.	If 'n' is a positive integer which is not a perfect square, then show that $\sqrt{n}$ is irrational.	CO1	8
		(OR)		
2.	a.	Given nonempty subsets A and B of **R**, let C denote the set $C=\{x+y:x\in A, y\in B\}$. Prove that if each of A and B has a supremum, then C has a supremum and $\sup C=\sup A+\sup B$.	CO1	10
	b.	State and prove Archimedean property of real number system.	CO1	10
3.	a.	Prove that the set of all real numbers is uncountable.	CO1	10
	b.	Let F be a collection of sets, then for any set B, we have i) $B-\bigcup\limits_{A\in F}A=\bigcap\limits_{A\in F}(B-A)$ ii) $B-\bigcap\limits_{A\in F}A=\bigcup\limits_{A\in F}(B-A)$	CO1	10
		(OR)		
4.	a.	Prove that every subset of a countable set is countable.	CO1	10
	b.	If F is a countable collection of disjoint sets, say $F=\{A_1,A_2,......\}$, such that each set A_n is countable, and then prove that the union $\bigcup\limits_{k=1}^{\infty}A_k$ is also countable.	CO1	10
5.	a.	Prove that a set S in R^n is closed, if and only if it contains all its adherent points.	CO2	8
	b.	State and prove Bolzano-Weierstrass theorem.	CO2	12
		(OR)		
6.	a.	If '$\bar{x}$' is an accumulation point of S, then prove that every n-ball $B(\bar{x})$ contains infinitely many points of S.	CO2	10
	b.	State and prove Cantor intersection theorem.	CO2	10
7.	a.	Let $f:S\to T$ be a function from one metric space (S,d_S) to another (T,d_T) and if $p\in S$, prove that f is continuous at p if and only if, for every sequence $\{x_n\}$ in S converges to p, the sequence $\{f(x_n)\}$ in T converges to $f(p)$.	CO3	10
	b.	State and prove Generalized Mean-Value Theorem.	CO3	10
		(OR)		
8.	a.	State and prove fixed point theorem for contractions.	CO3	10
	b.	State and prove Rolle's Theorem.	CO3	10

		Compulsory:		
9.	a.	Let each term of $\{f_n\}$ is a real-valued function having a finite derivative at each point on *(a, b)* and for at least one point x_0 in *(a, b)* the sequence $\{f_n(x_0)\}$ converges. And let there exists a function *g* such that $f_n' \to g$ uniformly on *(a,b)*. Prove that i) There exists a function *f* such that $f_n \to f$ uniformly on *(a,b)*. ii) For each *x* in *(a,b)* the derivative $f'(x)$ exists and equal to $g(x)$.	**CO3**	**20**

ALL THE BEST