Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – April / May– 2017**

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| **Code :** | **15MA3001** | **Duration :** | **3hrs** |
| **Sub. Name :** | **ALGEBRA** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | State and prove the divisional algorithm. | CO1 | 15 |
| b. | The Quadratic congruence x2+1  where P is an odd prime has a solution if and only if | CO1 | 5 |
| (OR) | | | | |
| 2. | a. | The linear Diophantine equation  has a solution if and only if d/c where d=gcd (a, b). If  is any particular solution of this equation, then all other solution are given by  ; , where t is an arbitrary integer. | CO1 | 10 |
| b. | Prove that if a|bc with gcd(a,b)=1, then a|c. | CO1 | 5 |
| c. | Find the gcd (475,120) and also write gcd as a linear combination of 475 and120. | CO1 | 5 |
| 3. | a. | Prove if P is a prime, then (P-1)! | CO1 | 10 |
|  | b. | If the integer a has order k modulo n, then  if and only if | CO1 | 5 |
|  | c. | Find all primitive roots of 9. | CO1 | 5 |
| (OR) | | | | |
| 4. | a. | Prove for any group G,  if and only if  and for a finite group, if and only if . | CO2 | 10 |
|  | b. | If  where P is a prime, then G is abelian. | CO2 | 5 |
|  | c. | Prove that a group of order 99 is not simple. | CO2 | 5 |
| 5. | a. | State and prove the Sylow theorem for abelian group. | CO2 | 10 |
|  | b. | Prove that a group of order 36 can not be simple. | CO2 | 10 |
| (OR) | | | | |
| 6. | a. | Prove that if G is a finite group and P is a prime number with Pn | O(G) and  O(G), then any two subgroup ofGof orderPn are conjugate. | CO2 | 10 |
|  | b. | Disscuss about the group of order 112132. | CO2 | 10 |
| 7. | a. | Prove that the set of all complex number J[i] is a Euclidean ring. | CO3 | 10 |
|  | b. | Prove that if R be an Euclidean ring, then any two elements a and b in R have a greastest common divisor d. Moreover d=λa+µb for some λ, µ ϵ R. | CO3 | 10 |
| (OR) | | | | |
| 8. | a. | Prove if R be an Euclidean ring and a,b ϵ R with b≠0 is not a unit element in R, then d(a) < d(ab). | CO3 | 10 |
|  | b. | Prove that if R be an Euclidean ring, then A=<ao> is a maximal ideal of R if and only if ao is a prime element of R. | CO3 | 10 |
|  | | **Compulsory:** |  |  |
| 9. | a. | State and prove Unique Factorization Theorem. | CO3 | 10 |
|  | b. | Prove that if R be a Euclidean ring, then every element in R is either a unit element or can be written as the product of finite number of prime elements of R. | CO3 | 10 |