**Reg. No. \_\_\_\_\_\_\_\_**

**Karunya University**

**(Karunya Institute of Technology and Sciences)**

(Declared as Deemed to be University under Sec.3 of the UGC Act, 1956)

**Supplementary Examination - June 2011**

**Subject Title: FOURIER SERIES, TRANSFORMS AND PARTIAL DIFFERENTIAL**

**EQUATIONS Time: 3 hours**

**Subject Code: MA247 Maximum Marks: 100**

#### **Answer ALL questions**

**PART – A (10 x 1 = 10 MARKS)**

1. Write the Fourier series of f(x) in (c, c+2𝓁) of periodicity 2𝓁.

2. If f(x) = x2 in (-π, +π) is expanded in Fourier series then what is the value of bn.

3. Solve: .

4. Form partial differential equation by eliminating constants from z = ax2 + by2.

5. Write the one dimensional wave equation.

6. Define steady state.

7. What is the suitable solution of steady state heat flow in two dimensions for the following boundary conditions? u(0, y) = 0 ; u(𝓁,y) = 0, u (x, 𝓁) = 0, u (x,0) = f(x); 0 ≤ x ≤ 𝓁.

8. In the study of two dimensional heat flow, for what type of plates are polar co-ordinates used?

9. If F{f(x)} = F(s), then F = \_\_\_\_\_\_\_\_.

10. If F{f(x)} = F(s), then F{eiax f(x)} = \_\_\_\_\_\_\_\_.

**PART – B (5 x 3 = 15 MARKS)**

11. Find the root mean square value of f(x) = x –x2 in – 1 < x < 1.

12. By eliminating arbitrary function, form partial differential equation of f (xyz, x + y + z) = 0.

13. The ends of a string of length ‘2𝓁’ are fixed and the midpoint of the string is taken to height ‘h’ and then released from rest in that position. Write the initial condition y (x, 0).

14. A rectangular plate is bounded by x = 0, x = a, y =0, y = b. Its surfaces are insulated and temperature along two edges y =0 and y = b are maintained at 60°C, while the other edges are kept at 0°C. Write the boundary conditions for the problem.

15. Find the finite Fourier cosine transform of f(x) = x in (0, π).

**PART – C (5 x 15 = 75 MARKS)**

16. a. Find the Fourier series of period 2π for the function f(x) = x (2π-x) in (0, 2π). Deduce the sum of the series, . (10)

b. Find half range cosine series of f(x) = (x -1)2 in 0 < x < 1. (5)

(OR)

17. a. Compute the first three harmonics of Fourier series of f(x) given by the following table: (10)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | π/3 | 2π/3 | π | 4π/3 | 5π/3 |
| f(x) | 0 | 9.2 | 14.4 | 17.8 | 17.3 | 11.7 |

b. Find half range sine series of f(x) = in (0, 𝓁). (5)

[P.T.O]

18. a. Solve: 2p+ 3q =1 b. Solve: z = px + qy + p2 – q2. c. Solve: p + x = qy (3 x 5)

(OR)

19. a. Solve:  b. Solve: 

c. Solve:  (3 x 5)

20. A tightly stretched string with fixed end points x= 0 and x = 60 is initially in a position given by 60x –x2. If it is released from rest from this position, find the displacement at any time and at any distance from the end x =0.

(OR)

21. A rod 20 cm long has its ends A and B kept at 40°C and 60°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u (x, t).

22. A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge y = 0 is u = 100sin3, 0 < x < 8 while two long edges x = 0 and x = 8 as well as the other short edge are kept at 0°C. Find steady state temperature distribution at any point of the plate.

(OR)

23. A semi circular plate of radius ‘a’ cm has insulated faces and heat flows in plane curves. The bounding diameter is kept at 0°C and the semi-circumference is maintained at temperature given by u = . Find the steady state temperature distribution at any point of the plate.

24. a. Find the Fourier transform of f(x) given by f(x) = and hence evaluate

i)  ii)  iii)  (12)

b. Solve for f(x) from the integral equation  (3)

(OR)

25. a. Find the Fourier sine and cosine transform of f(x) =  (12)

b. i) Find f(x) if its finite Fourier sine transform is for p = 1, 2, …., 0 < x < π.

ii) Find f(x) if its finite Fourier cosine transform is

Fc(p) = (3)